Supplementary Text #2

Plateaus by definition have a slope near zero, or at least clearly below one. In contrast, jumps in the curve have large slope values of >1. To identify plateaus and jumps in the curve of y: radiocarbon years against x: varve years in the Suigetsu record (Fig. 2), we developed a statistical method. It is based on the first derivative or slope of the curve over time, y'(x) = dy(x)/dx. This first-derivative method is an objective mathematical tool that can be also applied to other data series y(x), in this paper, to define slopes in the curves of y: radiocarbon years against x: core depth.

We use a running window, mathematically denoted as kernel, that is running along the x-axis and estimate the slope y' for x equal to the window's center using the points inside the window. This nonparametric technique is called kernel estimation, and using a smooth kernel function (such as an inverted parabola here employed) for weighting the points has the advantage of yielding a smooth estimate. For example, we may consider the case of the running mean for estimating the zeroth derivative or mean trend, which uses a non-smooth uniform kernel. The first derivative is estimated via the first derivative of the kernel's trend estimate. The kernel method can be adjusted at the boundaries of the x-interval by using a smaller kernel width ("boundary kernel"). Details of the kernel method for derivative estimation are mathematically described by Gasser and Müller (1979, 1984), and in a manner accessible to climatologists by Mudelsee (2014). Both these papers and the book also contain the mathematical formulas and further references. The software we developed for derivative estimation is provided as Fortran source code (kernel.f90) in the supplementary material.

To construct a 1-sigma error band around the slope curve, we used bootstrap resampling (Mudelsee, 2014). A resample y* is obtained by (1) calculating the residuals (the differences between trend and data points), (2) resampling pointwise random residuals, and (3) adding the random residuals back to the trend. In case of our data, autocorrelation effects (“memory” of x-values) were negligible, otherwise Step (2) could have been adapted by resampling blockwise. A new first derivative is estimated on the resample, yielding y*’(x). The procedure resampling–reestimation is repeated until B = 10,000 copies of y*’(x) are available. The standard error band results from the standard deviation over the B copies of y*’(x).

There is room for subjectivity in two dimensions. First, selection of the bandwidth (i.e., the width of the kernel function) determines the bias and variance properties of the slope estimate. There exist objective guidelines to solving this dilemma (Gasser and Müller 1984), but we aimed for a modest undersmoothing (smaller bandwidth) in order to “see” more fine details (at the cost of these details being less significant). Second, we selected a threshold value for defining a plateau or a jump. Adopting a threshold value of zero generated too many and too small-scale plateaus. A higher threshold value of 1.0 (i.e., one radiocarbon year per varve year) yielded clearly better agreement with the plateau boundaries previously determined by visual inspection. Furthermore, slope maxima refer to ^{14}C age jumps in the curve y(x), to be used for separating two adjacent plateaus. Finally, the 1-sigma uncertainty in varve years of the plateau boundaries was evaluated from the varve age, where the 1-sigma error band around the first derivative is crossing the threshold line of 1.

References

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Gasser, T., and Müller, H.-G. 1984. Estimating regression functions and their derivatives by the kernel method. *Scand. J. Stat.* **11**, 171–185.

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