‘TRANSITION DATING’ – A HEURISTIC MATHEMATICAL APPROACH TO THE COLLATION OF RADIOCARBON DATES FROM STRATIFIED SEQUENCES

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ABSTRACT. A heuristic approach, nicknamed “transition dating,” was used to date sequences of early Iron Age contexts using a series of 14C determinations. The basic principles of transition dating are simple and intuitive: 1) attempt to date transitions between periods, phases, etc. rather than the phases themselves, and 2) the most plausible date for that transition is one that is later than the dates from contexts preceding it, and is still earlier than the dates succeeding it. Hypotheses regarding the actual date of each transition may be evaluated using an appropriate loss function. These loss functions can also be adjusted or weighted by the user to account differentially for the various factors causing the distortion or “fuzz” in the dates.

THE PROBLEM

All archaeologists want to obtain precise, absolute dates for stratified deposits, whether excavating a site 10 millennia or 10 centuries old. Some years ago Idit Saragusti and I faced the problem of reconciling about 200 dates for the “late” periods at the site of Dor. These dates were obtained from various datable objects such as coins and wine-amphoras stamped with vintage dates. We developed a heuristic technique (Saragusti and Sharon 1995), which we dubbed “transition dating”. Here I present an adaptation of this technique to radiometric dating of earlier phases at the same site. This work is a part of the study conducted with Ayelet Gilboa, Israel Carmi, and Elisabetta Boaretto to investigate the historicity of the “united monarchy” of David and Solomon as described in the Bible (cf. in this volume: Gilboa 2001 and Sharon; Mazar and Carmi 2001; Bruins and van der Plicht 2001; for a recent book-size exposition see Handy 1997). The chronological dilemma is whether the archaeological horizon (designated hereafter “Ir2a”) attributed to this putatively historical period does indeed cover the 10th century BCE, or it is some 50–100 years later into the 9th century (Finkelstein 1996, 1998).

Figure 1 shows 22 14C determinations from a sequence of Early Iron Age horizons (labeled Ir1a, Ir1b, Ir1?|2?, and Ir2a phases) at the Phoenician (at that time) site of Dor. Full details of the contexts and their meanings are given in Gilboa’s paper, and will be dispensed with here. The dates are represented here in the form of probability distributions (after transformation to calendric time scale using the 1998 calibration curve). We have also noted for each sample a 67% and a 95% confidence interval, being the shortest contiguous interval accounting for 67% (95%) of the distribution’s weight. This is a slightly different estimate than the ones used in the OxCal (Bronk Ramsey 1995) or the Groningen CAL25 (van der Plicht 1993) programs, which allow for non-contiguous ranges. Nor is it synonymous with the straightforward transformation of the two-tailed 1σ or 2σ uncalibrated range into calendar years, which would not necessarily render the shortest possible span. At any rate, the algorithm developed herein works directly on the distribution curve, so the precise mechanics of estimating the curve’s variability are unimportant, except for illustrative purposes.

At first glance, these seem to represent a bewildering number of partly conflicting dates. This problem is all the more severe since one is dealing with a multi-phased site, where there are many differing dates from each of the different levels. Table 1 shows the full range of dates for each of the different phases, and we can see that the overlap is almost total. An average of all the dates per phase shows a chronological trend, but it is clearly unsatisfactory, too. A case where a sequence of phases yields overlapping or conflicting dating ranges will be called henceforward a “fuzzy” dating.
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**Figure 1** Distribution curves after calibration using the 1998 Groningen curve for 22 \(^{14}C\) dates from early Iron Age contexts at Dor.

**Legend for sample provenience exx:**
- RT2923 = Lab sample no. L7926 = Locus no. (Excavator’s context ID)
- B1/12 = Excavation area and local phase [i] context designation: [i] in situ; [p] other primary contexts such as trash pits, etc. [s] sealed deposits; less secure contexts were not used. “fire” = architectural complex/general stratigraphic context Ir1a–relative chronological horizon.
- \[^{14}C\] Measurement: 2875BP ± 25 = mean date in uncalibrated “radio-carbon years” before present, and standard deviation. Mean: 1020 BC = Mean calendaric date after calibration 95%: 1110–940 BC = 95% [or 67%] confidence interval (the shortest continuous span accounting for 95% [67%] of the distribution’s weight).
This work uses terms such as “fuzziness” or “ambiguity” in place of “probability”. The reason is that we are dealing with two distinct types of uncertainties here. Despite the intrinsically stochastic nature of the radioactive decay phenomenon, we can attach a definitive probability distribution to its occurrence. But what about the “probability” that a lab technician has contaminated a sample? Or that a field archaeologist retrieved the sample from an unrecognized rodent hole? We cannot really assess it, but hope and trust that such errors are avoided by most proficient practitioners most of the time. Bayesian statistics does sometimes incorporate these two types of uncertainties, by giving the latter type, too, an assumed or “a-priori” probability function. Most “classical” statisticians object to this, though, arguing that “belief” or “degree of expectation” are not the same as probability. The reasons for this fuzziness may further be categorized under two headings: those having to do with the nature of the samples themselves and the way they are measured, and those dependent on the nature of the contexts from which these samples come and the way they were excavated. In each of these categories we may isolate factors that would tend to produce “normal” dates clustering around the real date, and factors that tend to produce outliers of various kinds.

The Nature of the Samples

The typical standard deviations obtained were in the 25–40 year range. This spread is exacerbated by a factor of 0–50% by the calibration curve in the chronological range of the study. The sub-periods or “horizons” one can differentiate stratigraphically/typologically may be as short as 50 years or less. This means that in many cases the error spans of individual measurements will be wider than the entire span of the phase we are trying to date.

As we were forced to use long-lived samples where we did not have a sufficient sample size in short-lived ones, we must also contend with “old wood effects”. It is important to note that this type of error is one-sided, i.e. it can produce dates older than the real date of the deposit but not younger. We shall call such a phenomenon a residual date. Lastly, some error or “fuzz” could be introduced by lab procedures.

The Nature of the Deposits

Tell sites are composed primarily of anthropogenic fills. Primary (in situ) contexts, in which one assumes that all of the objects were in use at a single point in time, are often scarce. Fills contain material that has been used and discarded at any period within the “lifetime” of the phase it belongs to, as well as some out-of-context artifacts. Even if we limit ourselves to primary deposits, stratigraphic contemporaneity in the broad lateral expanses of tell sites should be regarded as broad rather than strict. The same is doubly true for sequences constructed by comparative artifact typology, i.e. the statement “Megiddo IVa and Tel Qasile X are contemporary” means that, at best, it can be shown

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Maximum span of 95% intervals</th>
<th>Maximum span of 67% intervals</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ir1a</td>
<td>1120 BC–800 BC</td>
<td>1080 BC–800 BC</td>
<td>950 BC</td>
</tr>
<tr>
<td>Ir1b</td>
<td>1160 BC–800 BC</td>
<td>1060 BC–820 BC</td>
<td>935 BC</td>
</tr>
<tr>
<td>Ir1?2?</td>
<td>1020 BC–800 BC</td>
<td>1000 BC–800 BC</td>
<td>870 BC</td>
</tr>
<tr>
<td>Ir2a</td>
<td>960 BC–800 BC</td>
<td>910 BC–820 BC</td>
<td>890 BC</td>
</tr>
</tbody>
</table>
according to the relative development of significant artifact-types that those of Megiddo IVA pre-
cede Qasile IX and are later than Qasile XI and vice versa. This is by no means the same as arguing
that Megiddo VIA and Qasile X were built and destroyed at the same precise moment in time. Thus,
when we speak of stratigraphic phases, or typological horizons, we must assume that each phase
represents a continuous occupation over time rather than a single episode. We must allow for a range
of different dates from each phase, representing the period in which this phase was in use. Dates
towards the end of a “phase” or “horizon” may be closer to the beginning of the next phase than to
those at the beginning of the same phase. Coupled with relatively long measurement spans, this may
lead to very similar-looking dates from different phases.

The assumption of occupational continuity must on no account be construed as license to assume
that the dates from this phase are expected to be uniformly distributed, as is sometimes done (e.g.
Manning and Weninger 1992), explicitly or implicitly. Indeed, the defining feature of architectural,
or tell stratigraphy is its punctuative nature, as opposed to the durative nature of most geological
types of deposition. Some episodes, such as construction and destruction, produce massive buildup
of deposits almost instantaneously, versus long periods of very little depositional activity. In the
same way, some depositional episodes (e.g. fires) are more conductive to the preservation of organic
materials than others. Thus, we should expect our 14C samples to be heavily skewed. The same is
true for the preservation of pottery or other artifacts that give relative dates to the phase.

Out-of-context artifacts do not date to the span in which the phase under discussion was occupied.
We divide these into two categories:

*Residual pieces* are early artifacts or ecofacts found in a (relatively) late stratum. These are quite
usual on tell sites. The very fact that people lived in a previously occupied site afforded ample
opportunity for early material to be mixed with contemporary discards. Walking on the tell today we
see any number of ancient artifacts scattered on the present-day surface, and this undoubtedly was
the case at any other point in its history. Thus, the assertion that all organic materials (even short-
lived ones) found in a closed or primary context must of necessity be in primary deposition is not
always warranted. Since most fills in built-up sites are intentional i.e. dirt dug up from one part of
the site in order to level up another, it is not surprising to see in them potsherds (as well as other
finds) from any period prior to the moment when the fill was deposited, though later periods (i.e. the
ones just prior to deposition) might be better represented.

*Intrusive pieces*, on the other hand, are late artifacts in an early context. Intrusions are the result of
post-depositional processes that caused disturbances unnoticed by the excavator, through which late
artifacts or ecofacts “filter” into early deposits. Sometimes these disturbances are very minor—such
as rodent burrows or root canals—and almost impossible to discern. Other times they are larger fea-
tures, overlooked by the excavator. At any rate, if the site was diligently excavated, these should be
rare. Note, again, the relative one-sidedness of contextual errors. Intrusion is less “natural” than
residuality and therefore one should expect more dates to be older than their contexts than vice-
versa. Finally, a context that is incorrectly phased by the stratigrapher will also cause its artifacts to
be considered out-of-context. The same is true for a misidentified artifact assemblage (due to insuf-
ficient sample size or investigator error), or for a misassociation of the context bearing the organic
sample(s) with the artifact assemblage context it is supposed to date (if the two are not one and the
same). Such mistakes might cause either a “residual” or an “intrusion-like” phenomenon.

Out-of-context dates are a pervasive predicament in any archaeological chronology, whether based
on coins or on wood (Saragusti and Sharon 1995). This difficulty can be reduced, but not completely
overcome, by greater care in excavation and recording, more discriminating choice of contexts and
samples, as well as more rigorous lab procedures (e.g. longer counting periods). Clearly, the practice of summarily discarding dates that are not to our liking carries with it the danger of bolstering our own preconceptions. Nowhere is this clearer than in the archaeological case in point. Dates incompatible with the normative “high” chronology for the Ir2a period have turned up over the years and were simply not published as they were considered “anomalous” (Finkelstein 1998).

A more justifiable procedure is possible in simulation-based algorithms such as the “Oxford Calibration Package”. If no simulated data set (or too few to form a sound sample) can be found to fit the model’s constraints and the observed measurements, the user may identify the offending observations and delete them from the model. Still, the basic answer most archaeologists seek is what date best fits the data with all the noise inherent in any real-life dating problem.

For the purposes of this study we used samples only from contexts whose placement in the stratigraphic sequence is unambiguous, with minimal post-depositional disturbance (primary deposits where we had them, and sealed accumulations otherwise). Only samples from contexts with representative artifact assemblages were used. Our primary consideration was to limit the measurement error. We therefore used the conventional radiometric counting technique (at the WISC 14 C facility) rather than the less exact accelerator mass spectrometry (AMS) method, limiting samples to those between 3 and 7 g. carbon after pretreatment, and subjecting them to 3000 minutes of counting each. Nevertheless, we do not make the assumption that all our samples are in-context.

THE PROPOSED SOLUTION

The challenge therefore is to utilize the relative advantage of the established stratigraphic/typological type-series dating in producing finely seriated sequences (the ones used in this case study are explicated in Gilboa 1999a,b and in this volume), in order to reduce the relative drawback of absolute 14 C dating. The fact that it does not produce a specific date to which the sample must belong, but rather “fuzzy” ranges, variegates chronological regions to which the sample could belong (at varying odds).

The basic tool we propose is the slight modification of an age-old principle used by historians and archaeologists—post quem/ante quem dating. It says that an event should be dated later than all datable objects drawn from contexts that can be shown to have been deposited prior to that event, and earlier than all dates from deposits definitely succeeding that event.

This sounds so simple as to be trivial. But it gets more complex because in order to propose a range for a whole sequence of ranges, our scheme would have to simultaneously propose a high and low date for each horizon, as well as avoid contradictions therein. We suggest a simple remedy for this. Rather than dating the deposits we will try to determine the dates of the transitions between them.

For each transition (noted henceforward by the symbol “|”, as in “Ir1a|b” or “Ir1|2”) we divide the entire sequence into context succeeding the transition and contexts preceding it (thus, a specimen from a context labeled “Ir2a” is later than the Ir1a|b transition). Reviewing the entire sequence when determining each transition enlarges the data base and ensures that there be no contradictions between the dates proposed for successive transitions. Such “splitting” of an ordinal data set to dichotomous sets of before/after categories is a standard practice in the analysis of ordinal data (Goodman 1984). In practice, it is not always possible to allocate each and every deposit to one of the two categories vis-a- vis a given interface, e.g. 14 C dates from a context labeled Ir1?/2? might not be useful for determining the date of the Ir1|Ir2 transition, but they may be used for dating the transition between Ir1a and Ir1b (in as much as they are definitely later than that transition) or between
the Ir2a and Ir2b. Therefore, for every transition we divided the entire data set to three categories: 

$^{14}$C dates from deposits above this interface, those from contexts below it, and those which are not relevant or from indeterminate contexts for this particular transition.

The dichotomous view of the data set necessitates some modification of our definitions of intrusion and residuality. We propose the following amendment: if the deposit in which an artifact was found is later than a given interface, but the artifact dates to before the proposed transition date, we shall call it residual (vis-a-vis this particular transition). If an artifact was found in a context earlier than the given transition, but is later than the proposed dating, we will consider it to be intrusive (Figure 2).

The case gets more complicated when, instead of unitary dates, one talks of ranges. Especially, when, as in the case of $^{14}$C dates, this range theoretically stretches from infinity to yesterday. We amend our definition again to search for a date later than the bulk of the distribution of as many as possible of the date ranges from contexts preceding it; but is still earlier than the bulk of the distribution of as many as possible of the date ranges from contexts succeeding it.

Here we should note a basic asymmetry, which holds both for what we have called specimen errors and contextual errors. “Old wood effect”, and contexts containing redeposited materials—i.e. what we have called “residual errors” are bound to happen in any tell. They are in a way “natural.” “Intrusion errors”—unnoticed pits, sample contamination, mis-attribution of artifacts, deposits or assemblages, etc.—always imply, to some extent, a failure on the part of the investigators. The textbook maxim that a context is dated by the latest find in it is a rephrasing of the post quem dating rule, which allows for redeposition, but completely ignores the possibility of intrusion errors. In real life, however, we must accede the fact that intrusion errors do occur, though they should be rarer than residual errors in any well-founded study.

**Mathematical Treatment**

The gist of our contention must be clear by now. We examine various possible hypotheses for the absolute dating of each transition in our stratigraphic/typological sequence. For each hypothesis we note the number of “in context” versus “out of context” dates, dividing the latter to “residual dates” and “intrusive dates.” In the quantitative case we examine the degree of “residuality” and “intrusiveness” for each hypothesis. The “best guess” as to each transition’s date is the one that would mini-
mize both for a given dataset. In cases where lowering the residuality would raise intrusiveness and vice-versa, lowering the intrusiveness should be preferred.

In order to assess the relative merit of opposing hypotheses (e.g., for the case in point: “The Ir1|2 transition occurs at 1000 BCE”—e.g. Mazar 1990: 296; versus “it is at 925 BCE”—Finkelstein 1996) we must construct an objective function which would assign a “grade” to any possible hypothesis, or a loss function which would assign “zero loss” to a hypothesis producing no “error” for a given data set, and progressively higher values to hypotheses, in which many “anomalies” need to be explained. We do so heuristically, quite peremptorily building a mathematical expression, which would behave in the manner described below:

Let \( B|A \) denote a relative-dating transition defined on stratigraphical and/or typological grounds, where \( B \) are all contexts before the transition, \( A \) are all contexts after it, and \( p(t) \) denote the calibrated probability for a sample to date to time \( t \). If \( j \) is a \(^{14}\)C determination from a context \( A \) i.e. later than \( B|A \), we shall refer to the expression:

\[
f_j(t) = \int_{-\infty}^{t} p(t)dt
\]

as “the residual error” of sample \( j \) under the hypothesis \( H: \text{Date}(B|A) = t \).

Similarly, If \( k \) is a \(^{14}\)C determination from a context \( B \), the expression will be referred to as “the intrusion error” of sample \( k \):

\[
f_k(t) = \int_{t}^{\infty} p(t)dt.
\]

We have used the following “family” of loss functions:

\[
F_{B|A}(t) = \left(1 + \sum_{j \in A} \left[f_j(t)\right]^R\right)^r + \left(1 + \sum_{k \in B} \left[f_k(t)\right]^I\right)^i.
\]

\( R \) and \( r \) are user-supplied constant parameters (different values of which make \( F \) a “family” rather than a unitary function). They control the relative “penalty” incurred for intrusion versus residual errors.

High values for \( i \) and \( r \) would tend to place more weight on relatively large intrusion/residual errors. It must be remembered that \( p(t) \) is defined for \(-\infty \) to 0 BP. Thus even a sample by and large preceding a given transition date will have a long, thin “tail” on the positive side of the transition, and vice-versa. It is usually in the user’s favor to downplay such “tails” vis-a-vis more substantive “errors”.

\( R \) and \( I \) are weights applied to differentiate the total intrusions vs. total residuals. Since the expressions in round parentheses are constrained to values greater than 1, using higher values for \( I \) than for \( R \) will “penalize” intrusive dates more than residual dates. If \( R=1 \) and \( I=2 \), one intrusion will be “equivalent” to two residuals, 2 intrusions to 8 residuals, etc. Figure 3 shows several functions, with different parameter values, calculated for the end of the Iron I period (i.e. what we have called here the transition between Ir1b and Ir1b?2? Horizons). As expected, the best-fit transition date creeps upwards the higher the “punishment” for intrusion, relative to residuality.
Solving the function $F$ for all $t$'s and/or locating the $t$ which minimizes it will give us the most reasonable transition date between $B$ and $A$. It would be possible, of course, to use a function-minimization algorithm to calculate the exact extremum point of $F(t)$, but usually a visual inspection of the graphs (calculated here at 10 calendaric year intervals) is sufficient. The three graphs at the top of Figure 4 give the "loss values" for the transitions between Ir1a and Ir1b; Ir1b and Ir1|2; Ir1|2 and Ir2a using the above loss function with $i=r=4$, $I=2$, $R=3$.

There are any number of other functions that would behave similarly. Another example is:

$$G(t) = \prod_{j \in A} \int_{t}^{\infty} p(t) dt \prod_{j \in B} \int_{-\infty}^{t} p(t) dt.$$

This is an "objective" function and the best-fit model will give it a maximum value. The above expression denotes the probability that all the dates from post-transition context are in fact later than the hypothesized $t$ (data set contains no residuals), while all dates from pre-transition contexts are earlier than $t$ (no intrusions), under the (arguable) assumption that all our samples denote statistically-independent random events. Heuristically, though, this function would also behave in the "proper" manner—give higher values the less intrusions and residuals are posited by a hypothesis $H$: $t$ for a given data set. In this case, too, we can weight the two multiples to give preference to residual errors over intrusions. One computational difficulty that must be avoided is computer underflow errors as a result of multiplying many values close to zero, which would be the case if there are many intrusions and residuals in the data set.

**DISCUSSION**

We have found that when the data set is robust, most reasonable functions with reasonable parameter values give similar results. The example in Figure 4 shows that all four functions cluster around the 900–850 BCE range. Indeed, testing each transition with several different loss functions can give a good estimate of the robustness of the data set. Chaotic ones will tend to vary widely with small changes to the parameters of the loss function.
“Transition dating” is not, then, a singular method giving one unique result to each data set. Rather, we advocate looking at each data set from several different angles and exploring how the transition dates might change under different assumptions. Indeed, each problem and each data set is different. Sites where the primary formation process is human construction should not be treated the same as those geologically deposited. Samples collected from constructional fills should be weighted differently than those originating in primary deposits. A specimen consisting of a collection of charcoal fragments is not the same as a single charred grain when considering the chances of residuality versus intrusion. While greatly reducing the noise in the initial data set, “transition dating” itself is fuzzy. But then so are the transitions we set out to measure; note the use of “indeterminate” categories such as “Ir1|2” within the cultural horizons in our data set.

The “Oxford Calibration Package” (Bronk Ramsey 1994, 1995, 2001) contains routines that allow various constraints to be put upon a data set, including dividing it into phases and calculating beginning and end distributions for each phase using Bayesian statistics. Why, then, reinvent the wheel? For one thing, “transition dating” has the merit of closely emulating the type of logic archaeologists have always used to date their deposits. It is also computationally simple—all graphs and computations herein were done with an Excel spreadsheet. Moreover, transition dating and Bayesian statistics are as different as can be in their guiding “Zen” and primary assumptions. In this sense, the two methods should be regarded as complementary. Any model, in archaeology and in statistics, has preliminary assumptions that should be questioned, and parameters which should be decided on or estimated. The more elaborate the model, the more assumptions it makes, and the more it is dependent on these assumptions. Therefore, should the results of two very different models agree, one can with some confidence claim that they are not an artifact of the initial assumptions. If Bayesian estimation and “transition dating” give very different results, the data set is probably not very robust and the results are suspect. For the Dor case, suffice it to say that the results obtained by using the “Oxford Calibration Package” were always within the cluster of results found by trying different parameters on the loss functions.

RESULTS AND CONCLUSION

According to the Tel Dor data set of 22 14C dates, the transitions within and between the Iron I period for Phoenicia are as shown in Figure 4:

Ir1a|b: The most plausible point for the transition between Ir1a and Ir1b is 970–920 BCE, depending on the loss function parameters and general “flatness” of most curves in that range. To err on the
side of caution we date the Ir1a|b transition to circa 975 BCE. The conventional date for this transition has hitherto been circa 1050 BCE.

Ir1b|Ir2: The transition between the Ir1b and the “transitional” phase that we have labeled at Dor “Ir1?/2?” is somewhere between 880 and 860 BCE. This transitional phase ends circa 850–820. Again, to err only on the side of caution, we may suggest a round date of around 850 BCE for the beginning of Ir2a, conventionally dated at 1000 BCE.

Thus, the Dor data supports a “super-low” chronology for the Levantine Iron Age. The implications of this chronology are discussed by Gilboa (this volume).

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