

FORMATION OF $^{13}\text{C}/^{12}\text{C}$ ISOTOPE RATIOS IN SPELEOTHEMS: A SEMI-DYNAMIC MODEL¹

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ABSTRACT. A theoretical approach towards predicting the carbon isotope composition of carbonate cave deposits is presented. The proposed model simulates time variations of both the chemical and isotopic composition of the carbonate solution and deposited calcite in the course of CaCO_3 precipitation. Two phases of the precipitation process are distinguished and treated separately: initial outgassing of the solution until a certain degree of supersaturation of CO_3^{2-} ions is reached and subsequent precipitation of CaCO_3 related to further outgassing. Precipitation rates of CaCO_3 predicted by the model agree fairly well with literature data. The model predicts $\delta^{13}\text{C}$ values of deposited calcite within a range of ca -16 to $+3\text{‰}$ depending on temperature, chemical and isotope parameters of the initial solution and actual degree of the precipitation process.

INTRODUCTION

Speleothems are deposits of calcium carbonate found in limestone caves (stalagmites, stalactites and flowstones). They are formed by precipitation from seepage water entering through the roof or walls of the cave which becomes supersaturated with CaCO_3 . Several models describing the evolution of the carbon isotope composition of precipitated carbonates have been proposed (eg, Hendy 1971; Dreybrodt 1982; Salomons & Mook 1986).

The most comprehensive analysis of the CaCO_3 precipitation process was made by Hendy (1971). He describes mathematically CaCO_3 precipitation under equilibrium conditions for both open and closed systems. He also presents first attempts towards kinetic treatment of isotopic exchange in the reaction chain $\text{CO}_{2g} \rightleftharpoons \text{CO}_{2w} \rightleftharpoons \text{HCO}_3^-$ and formulates physical and chemical conditions to be fulfilled for equilibrium fractionation of carbon and oxygen isotopes during speleothem formation processes.

Salomons and Mook's (1986) theoretical approach to the formation of ^{13}C content of precipitated carbonate is in fact, very similar to Hendy's model, with some simplifications.

Both these models assume that outgassing of the solution and precipitation of CaCO_3 occur simultaneously from saturated solution. They also assume that isotopic equilibrium is maintained between gaseous and dissolved CO_2 .

Dreybrodt's model (1982) refers to specific conditions in caves situated under glaciers or in bare karst areas where precipitation of CaCO_3 may be due to geothermal effect. This model was proposed to explain very high $\delta^{13}\text{C}$ values of speleothems (up to $+10\text{‰}$) from the high mountain caves of the Alps (Geyh, Franke & Dreybrodt 1983).

Recent isotope investigations of streams precipitating CaCO_3 revealed that deposition of CaCO_3 starts, in fact, from supersaturated solutions, with activity of CO_3^{2-} ions equal to ca $2.2 \cdot 10^{-5}$ mol/L (Usdowski, Hoefs & Menschel 1979; Dandurand *et al* 1982; Michaelis, Usdowski & Menschel 1985). Also, the assumption of full chemical and isotopic equilibrium in the solution cannot be strictly valid because the rate of removal of CO_2 from the solution, especially at initial stages of the outgassing process, is much larger than its production rate of HCO_3^- ions. Consequently, kinetic isotope relationships rather than equilibrium should be used. Our model will hopefully eliminate many simplifying assumptions made in the above-mentioned models.

¹ This paper was presented at the 13th International Radiocarbon Conference, June 20-25, 1988, in Dubrovnik, Yugoslavia.

THE H₂O-CO₂-CaCO₃ SYSTEM

The H₂O-CO₂-CaCO₃ system, under full chemical equilibrium conditions can be described by the following equations (Buhmann & Dreybrodt 1985)

$$(H^+)(OH^-) = K_W(H_2O) \quad (1a)$$

$$(CO_2) = K_H P_{CO_2} \quad (1b)$$

$$(CO_2) = K_O(H_2CO_3^0) \quad (1c)$$

$$(H^+)(HCO_3^-) = K_1(H_2CO_3^*) \quad (1d)$$

$$(H^+)(CO_3^{2-}) = K_2(HCO_3^-) \quad (1e)$$

$$(H^+)(HCO_3^-) = K_4(CO_2)(H_2O) \quad (1f)$$

$$(H^+)(HCO_3^-) = K_3(H_2CO_3^0) \quad (1g)$$

$$(Ca^{2+})(CO_3^{2-}) = K_C(CaCO_3^0) \quad (1h)$$

where $(H_2CO_3^*) = (H_2CO_3^0) + (CO_2)$, K_H – solubility of CO₂ in water and the expressions in brackets represent activity of individual ions in the solution. The presence of CaCO₃⁰ and CaHCO₃⁺ molecules can be ignored because of their low concentration in the solution, usually <1% of the total amount of all other carbonic components (Langmuir 1984). Also, the condition of electroneutrality of the solution should be fulfilled

$$c_H + 2c_{Ca} - c_{OH} - c_{HCO_3^-} - 2c_{CO_3^{2-}} = 0 \quad (2)$$

where c represents the concentration of the given ion in the solution (in mol/L). The equilibrium constants quoted in Eqs (1a)–(1h) are temperature-dependent (see eg, Buhmann & Dreybrodt 1985).

In a natural environment, limestone dissolution occurs usually with participation of biogenic CO₂ originating from plant respiration and decay of dead organic matter. The soil layer is the principal reservoir of biogenic CO₂ where its partial pressure can be up to two orders of magnitude higher than in the bulk atmosphere.

Limestone dissolution may proceed along two extreme routes:

1) under open-system conditions where the solution is always in contact with the reservoir of CO₂,

2) under closed-system conditions where the solution is isolated from the reservoir of gaseous CO₂ before dissolution begins. In nature, the dissolution process mostly should be considered as a mixture of these extreme situations.

In an open system, with chemical and isotopic equilibrium between the CO₂ reservoir and the solution, the chemical composition of the solution can be determined from Eqs (1a)–(1h) and (2). The carbon isotopic composition of individual ions in the solution can be calculated using the equilibrium isotopic enrichment constants listed in Table 1. It is evident that, under open-system conditions, the isotopic composition of all carbonic compounds present in the solution is controlled by the isotopic composition of the gaseous CO₂.

Under closed-system conditions, carbon atoms in the solution originate from two sources, the soil CO₂ and dissolved limestone rock. Again, assuming full chemical and isotopic equilibrium in the solution, the carbon isotope composition of individual carbonate compounds in the solution can be calculated for each step of the dissolution process.

Figure 1 shows the evolution of the isotopic composition of HCO₃⁻ ions and total dissolved inorganic carbon (TDIC) in the solution during dissolution, for values of temperature and partial pressure of CO₂ typical for cave environments. The calculations were based on the iterative method proposed by Hendy (1971). We assumed the $\delta^{13}C$ of soil CO₂ equal

TABLE 1
Equilibrium isotope enrichment coefficients $\epsilon_{\text{A-B}}$ (‰) for the $\text{CO}_2\text{-H}_2\text{O-CaCO}_3$ system at 5° and 15°C. Data after Mook, Bommerson & Staverman (1974), Vogel, Grootes & Mook (1970), Thode, Shima, Rees & Kirshnamurthy (1965), Salomons & Mook (1986).

TEMPERATURE 5 °C						
A	B	CO_{2g}	CO_{2w}	HCO_3^-	CO_3^{2-}	CaCO_3
CO_{2g}		+	+ 1.15	-10.20	- 9.61	-10.09
CO_{2w}		- 1.15	+	-11.35	-10.74	-11.24
HCO_3^-		+10.31	+11.48	+	+ 0.60	+ 0.11
CO_3^{2-}		+ 9.70	+10.86	- 0.60	+	- 0.49
CaCO_3		+10.19	+11.37	- 0.11	+ 0.49	+
TEMPERATURE 15 °C						
A	B	CO_{2g}	CO_{2w}	HCO_3^-	CO_3^{2-}	CaCO_3
CO_{2g}		+	+ 1.10	- 9.02	- 8.54	- 9.43
CO_{2w}		- 1.10	+	-10.12	- 9.63	-10.53
HCO_3^-		+ 9.10	+10.22	+	+ 0.49	- 0.41
CO_3^{2-}		+ 8.61	+ 9.72	- 0.49	+	- 0.90
CaCO_3		+ 9.52	+10.64	- 0.41	+ 0.90	+

to -25‰ *vs* PDB which is the mean value occurring on areas covered by C_3 plants. The $\delta^{13}\text{C}$ of limestone carbonate was fixed at the level of $+1\text{‰}$ *vs* PDB. As shown in Figure 1, the $\delta^{13}\text{C}_{\text{TDIC}}$ values at the saturation point vary between -15‰ and -19‰ under open-system conditions, whereas the corresponding values for the closed system are in the range of -13‰ to -15‰ . The corresponding $\delta^{13}\text{C}$ values for HCO_3^- ions in the solution lie in the range of -14.9‰ to -16.1‰ and -13‰ to -14‰ , for open- and closed-system conditions, respectively. There is only a slight dependence of $\delta^{13}\text{C}_{\text{HCO}_3^-}$ on temperature because changes of equilibrium fractionation factors with temperature are compensated by parallel changes of the molar ratios of carbonic molecules in solution.

FORMATION OF SPELEOTHEMS

Under natural conditions, calcite speleothems are formed when a thin film of carbonate solution depositing CaCO_3 flows down the given specimen. For a theoretical description of this process, a planar model has been proposed (Fig 2). The liquid film is assumed to be ca 0.01cm thick, which is the value most frequently observed in nature. A constant pH value across the film was further assumed (Hoover & Berkshire 1969) which considerably simplifies numerical calculations. In view of the isotope and chemical data for carbonate precipitating springs (*cf* Introduction), the formation process of speleothems was divided into two parts: 1) outgassing of the solution without precipitation of CaCO_3 until a certain degree of supersaturation of the solution is reached, 2) simultaneous outgassing of the solution and precipitation of CaCO_3 . In both cases, the $\delta^{13}\text{C}_{\text{TDIC}}$ and $\delta^{13}\text{C}$ of precipitated CaCO_3 were calculated as functions of the time elapsed since the beginning of the outgassing process.

Outgassing of the Solution

During the release of CO_2 from the solution, the conversion rate of HCO_3^- to CO_{2w} is usually much faster than the backward reaction. Thus, the reaction $\text{CO}_{2w} \rightarrow \text{HCO}_3^-$ was omitted from the calculations. Further, we assumed that the solution preserves the initial

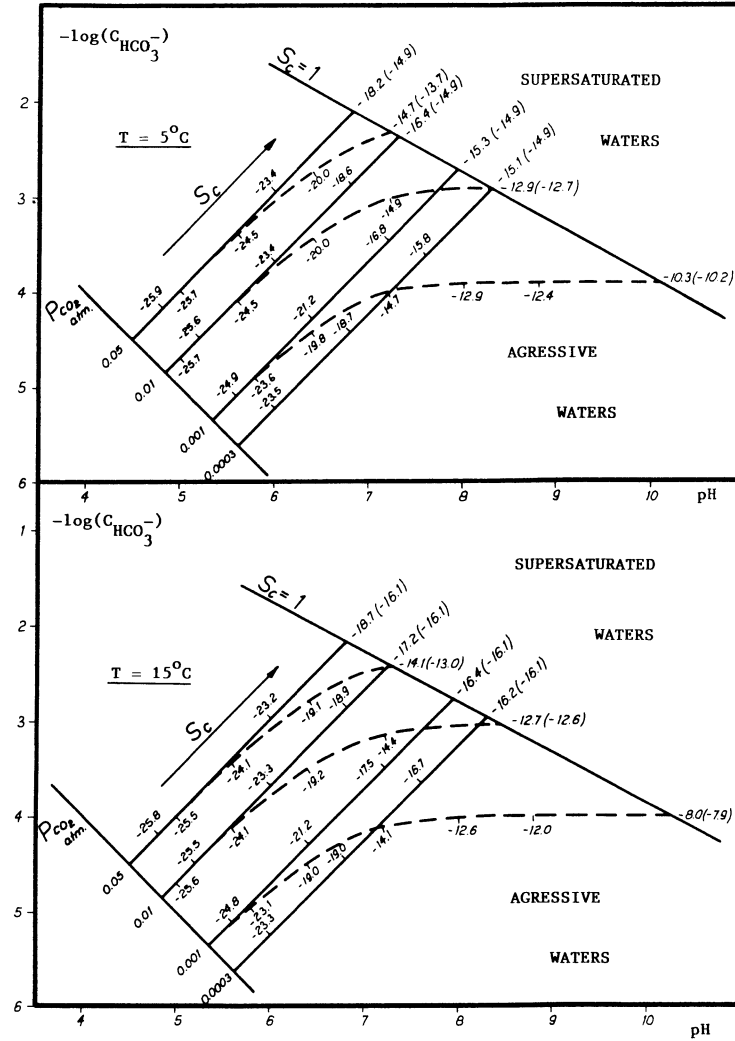
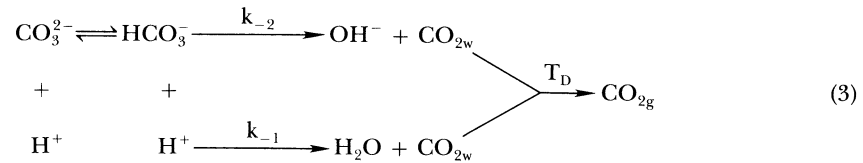


Fig 1. Dissolution of CaCO_3 in water under closed- (----) and open- (—) system conditions at $+5^\circ$ and $+15^\circ\text{C}$. Numbers labeling the curves indicate $\delta^{13}\text{C}$ values of the TDIC. Numbers in parentheses are $\delta^{13}\text{C}$ values of HCO_3^- in the saturated solution ($S_c = 1$). The input $\delta^{13}\text{C}$ values are -25 and $+1\text{‰}$ vs PDB for soil CO_2 and limestone carbonate, respectively.

concentration of Ca^{2+} ions during removal of CO_2 . Based on these assumptions, the outgassing of the solution can be described by



where the reaction $\text{CO}_3^{2-} + \text{H}^+ \rightleftharpoons \text{HCO}_3^-$ is fast (Dreybrodt 1980). Conversion of HCO_3^- to CO_{2w} can be treated as a simultaneous, pseudo-first-order reaction. Consequently, temporal

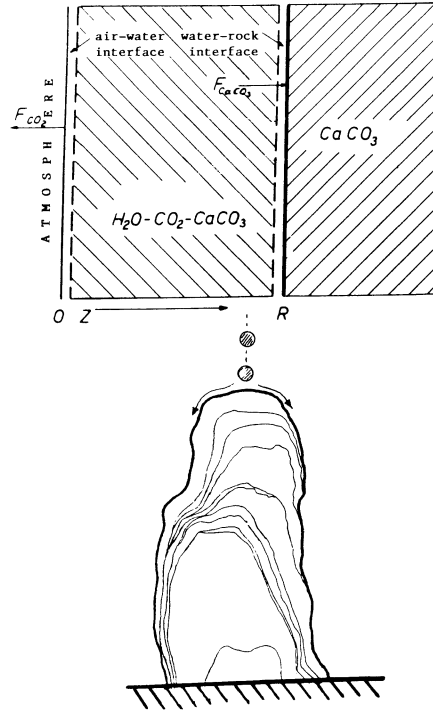


Fig. 2. Schematic representation of processes on the surface of the growing speleothem

variations of concentration of HCO_3^- ions in the solution can be expressed by

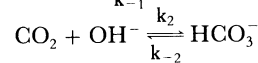
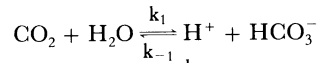
$$c_{\text{HCO}_3^-}(t) = c_{\text{HCO}_3^-}^{\text{atm}} + \{c_{\text{HCO}_3^-}^0 - c_{\text{HCO}_3^-}^{\text{atm}}\} \text{EXP}(-k_{\text{HCO}_3^-} t) \quad (4)$$

where $c_{\text{HCO}_3^-}(t)$, $c_{\text{HCO}_3^-}^0$, $c_{\text{HCO}_3^-}^{\text{atm}}$ represent concentrations of HCO_3^- ions at the time instant t , $t = 0$ and in equilibrium with the atmospheric CO_2 , respectively. The rate constant $k_{\text{HCO}_3^-}$ in Eq (4) depends on the pH of the solution and is equal

$$k_{\text{HCO}_3^-} = k_{-2} + k_{-1}c_{\text{H}^+} \quad (5)$$

where k_{-2} and k_{-1} are the rate constants of the reactions shown in Eq (3). Numerical values of k_{-2} and k_{-1} are listed in Table 2.

TABLE 2
Chemical rate constants for the reactions



(according to Buhmann & Dreybrodt 1985)

Temperature [°C]	k_1 [1/s]	k_2 [L/mol*s]	k_{-1} [L/mol*s]	k_{-2} [1/s]
0	$2.3 \cdot 10^{-3}$	995	6966	$4.4 \cdot 10^{-6}$
5	$4.2 \cdot 10^{-3}$	1574	11096	$9.7 \cdot 10^{-6}$
10	$7.5 \cdot 10^{-3}$	2449	17673	$2.1 \cdot 10^{-5}$
15	$1.3 \cdot 10^{-3}$	3753	28136	$4.4 \cdot 10^{-5}$

We further assumed that diffusion is a principal mechanism responsible for transport of CO_{2w} across the film thickness. The solution of a one-dimensional diffusion equation with a homogeneous source of CO_{2w} (reservoir of HCO_3^- ions), for the time intervals fulfilling the condition $D_{\text{CO}_2} t/R^2 > 0.4$, can be expressed by

$$c_{\text{CO}_{2w}}(t) = \frac{8}{\pi^2} \left(\text{EXP}(-t/T_D) \{c_{\text{CO}_{2w}}^o - c_{\text{CO}_{2w}}^{\text{atm}}\} + \frac{\{c_{\text{HCO}_3^-}^o - c_{\text{HCO}_3^-}^{\text{atm}}\} T_D k_{\text{HCO}_3^-}}{1 - T_D k_{\text{HCO}_3^-}} * \right. \\ \left. * \{ \text{EXP}\{-k_{\text{HCO}_3^-} t\} - \text{EXP}\{-t/T_D\} \} \right) + c_{\text{CO}_{2w}}^{\text{atm}} \quad (6)$$

where $c_{\text{CO}_{2w}}(t)$, $c_{\text{CO}_{2w}}^o$ and $c_{\text{CO}_{2w}}^{\text{atm}}$ are the average concentrations of CO_{2w} in the liquid film at the time t , at $t = 0$ and in equilibrium with atmospheric CO_2 , respectively. T_D stands for characteristic constant of CO_2 decrease in the solution and is expressed as

$$T_D = \frac{4R^2}{\pi^2 D_{\text{CO}_2}} \quad (7)$$

where R is the film thickness (in cm) and D_{CO_2} is the diffusion coefficient of CO_2 in water (in cm^2/s). The temperature dependence of D_{CO_2} was adopted after Buhmann & Dreybrodt (1985).

The net mass and isotope fluxes of CO_2 through the air-water interface can be expressed as a difference between corresponding fluxes “out” and “into” the solution (Siegenthaler & Munnich 1980)

$$F = F_{\text{as}} - F_{\text{sa}} = w * (K_H P_{\text{CO}_2} - c_s) \quad (8)$$

$$F^{13} = w^{13} \alpha_{\text{CO}_{2g}-\text{CO}_{2w}} K_H^{13} R_a P_{\text{CO}_2} - w^{13} \alpha_{\Sigma-\text{CO}_{2w}}^{13} R_{\Sigma} c_s \quad (9)$$

where

- F, F^{13} = net fluxes of $^{12}\text{CO}_2$ and $^{13}\text{CO}_2$, respectively
- $w, ^{13}w$ = transport velocities of $^{12}\text{CO}_2$ and $^{13}\text{CO}_2$ molecules through the liquid film
- $^{13}R_a, ^{13}R_{\Sigma}$ = $^{13}\text{C}/^{12}\text{C}$ isotope ratios of atmospheric CO_2 and TDIC in solution
- P_{CO_2} = partial pressure of atmospheric CO_2
- K_H = solubility of CO_2 in water
- $\alpha_{\text{CO}_2-\text{CO}_{2w}}$ = equilibrium fractionation factor between gaseous and dissolved CO_2
- $\alpha_{\Sigma-\text{CO}_{2w}}$ = effective equilibrium fractionation factor between TDIC and CO_{2w} in solution
- c_s = concentration of CO_{2w} in bulk solution.

Dividing Eq (9) by Eq (8) and transforming the isotopic ratios R into corresponding δ values we obtain

$$\delta^{13}C_F = \left(\frac{\alpha_{\text{as}} c_{\text{CO}_2}^{\text{atm}} \{\delta^{13}C_{\text{CO}_2}^{\text{atm}} 10^{-3} + 1\}}{K_H P_{\text{CO}_2} - c_{\text{CO}_{2w}}} - \frac{\alpha_{\text{sa}} c_{\text{CO}_{2w}} \{\delta^{13}C_{\Sigma} 10^{-3} + 1\}}{K_H P_{\text{CO}_2} - c_{\text{CO}_{2w}}} - 1 \right) * 1000 \quad (10)$$

where $\alpha_{\text{as}} = \alpha_{\text{CO}_{2g}-\text{CO}_{2w}} * ^{13}w/w$ and $\alpha_{\text{sa}} = \alpha_{\Sigma-\text{CO}_{2w}} * ^{13}w/w$. The above formula represents the $\delta^{13}\text{C}$ value of the net flux of CO_2 through the interface.

Calculations of the evolution of $\delta^{13}C_{\text{TDIC}}$ during the outgassing process started from the parameters of the solution saturated with CaCO_3 . Based on Eqs (4) and (6), for the time step Δt , the actual concentrations of CO_{2w} and HCO_3^- ions were calculated. Assuming chemical activity coefficients of all carbonic molecules equal 1, we evaluated the concentration of hydrogen ions H^+ using conditions of electroneutrality and the assumption of chemical

equilibrium between HCO_3^- and CO_3^{2-} ions. After computation of the concentrations of other ions in solution, the ionic strength was used (Garrels 1960) to calculate activity coefficients, which in turn, were applied to calculate the concentrations of carbonic compounds in solution more accurately. The iterative process was repeated until successive changes in activity coefficients were $<10^{-4}$. The values of Δc_{TDIC} , $\delta^{13}\text{C}_{\text{TDIC}}$, and isotopic composition of all carbonic compounds remaining in the solution were then computed. Since the kinetic fractionation factor for the $\text{HCO}_3^- \rightarrow \text{CO}_{2\text{w}}$ reaction is unknown, the equilibrium values have been adopted. The computation was performed for successive intervals Δt , until the activity of CO_3^{2-} ions necessary for initiation of CaCO_3 precipitation process was reached ($\sim 2.2 \cdot 10^{-5}$ mol/L).

The degassing process was simulated for four carbonate solutions with different initial parameters. The results are summarized in Figure 3. The initial, fast increase of $\delta^{13}\text{C}_{\text{TDIC}}$ is caused by release from the solution of large amounts of $\text{CO}_{2\text{w}}$ with low ^{13}C content. A further increase of $\delta^{13}\text{C}_{\text{TDIC}}$ proceeds slowly because the $\text{CO}_{2\text{w}}$ reservoir is exhausted and the outgassing rate is controlled by the HCO_3^- ions. At 5°C , after the first 20 sec, the HCO_3^- ions entirely control the outgassing process. The influence of atmospheric CO_2 on the isotope composition of the solution is negligible because the flux of CO_2 "out" of the solution is much larger than the backward flux. In Figure 3 it becomes clear that solutions of similar origin (closed or open system), start to precipitate calcite earlier for higher partial pressures of CO_2 at the time of their formation.

Temperature is an essential parameter accelerating the outgassing process. An increase in temperature slightly decreases the difference in $\delta^{13}\text{C}_{\text{TDIC}}$ between saturated and outgassed solutions. As in Figure 3, this difference is in the range of 3-4‰. It should be noted that final values of $\delta^{13}\text{C}_{\text{TDIC}}$ may in reality be slightly lower because in the calculations we assumed isotopic equilibrium between carbonate compounds in solution. Escape of CO_2 to the atmosphere is a fast process and the HCO_3^- ions probably are not able to reach equilibrium with CO_2 remaining in solution.

Precipitation of CaCO_3

When the activity of CO_3^{2-} ions in solution reaches ca $2.2 \cdot 10^{-5}$ mol/L, both outgassing and precipitation of CaCO_3 proceed simultaneously. It is a well-known experimental fact that precipitation of 1 mole of CaCO_3 is accompanied by release of 1 mole of gaseous CO_2 to the atmosphere (Michaelis, Usdowski & Menschel 1985). We numerically simulated the precipitation process in a similar way to outgassing the solution, described above. The calculation started from outgassed, supersaturated solution with defined chemical and

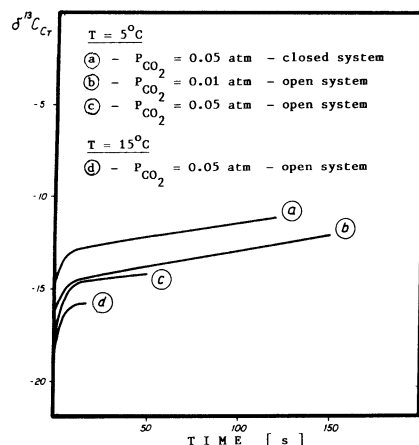


Fig 3. Evolution of $\delta^{13}\text{C}$ values of the TDIC in solution during initial outgassing without precipitation of CaCO_3 . The activity of CO_3^{2-} ions in solution, corresponding to final points of the curves, equals ca $2.2 \cdot 10^{-5}$ mol/L.

isotopic composition. The time evolution of $\delta^{13}\text{C}_{\text{TDIC}}$ was computed by an iterative method, using the isotope mass balance formula

$$\delta^{13}\text{C}_{\text{TDIC}}^{i+1} = \delta^{13}\text{C}_{\text{TDIC}}^i \frac{c_{\text{TDIC}}^i}{c_{\text{TDIC}}^{i+1}} - \delta^{13}\text{C}_{\text{F}}^i \frac{\Delta c_{\text{TDIC}}}{2 c_{\text{TDIC}}^{i+1}} - \delta^{13}\text{C}_{\text{CaCO}_3}^i \frac{\Delta c_{\text{TDIC}}}{2 c_{\text{TDIC}}^{i+1}} \quad (11)$$

where $\delta^{13}\text{C}_{\text{TDIC}}^{i+1}$ represents the carbon isotopic composition of the precipitated portion of CaCO_3 , corresponding to the change in the total carbon content of the solution c_{TDIC}^i by Δc_{TDIC} . Isotopic equilibrium between HCO_3^- and CO_3^{2-} ions in the solution was assumed (high rate of $\text{HCO}_3^- \rightarrow \text{CO}_2 + \text{H}^+$ reaction). The isotopic composition of total precipitated calcite ($\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$) was calculated as a weighted mean of the $\delta^{13}\text{C}_{\text{CaCO}_3}^i$ values. Results of the calculations are summarized in Figure 4, from which several interesting conclusions about the dynamics of the precipitation process can be drawn:

1) The ^{13}C content of individual portions of CaCO_3 increases strongly during the initial stages of precipitation. This is mainly due to an ongoing outgassing process accompanied by a relatively large isotope fractionation factor for the $\text{HCO}_3^- \rightarrow \text{CO}_2$ reaction which should be compared with the small value of $\epsilon_{\text{CaCO}_3-\text{HCO}_3^-}$ (cf Table 1). This "two-sink," Rayleigh-type process results in values which, for a certain interval, may be even higher than equilibrium values with atmospheric CO_2 . Such an "inertia" effect can be seen especially well in the $\delta^{13}\text{C}_{\text{CaCO}_3}^i$ curve calculated for 15°C . In later stages of precipitation, when the P_{CO_2} in solution is only 10–20% higher than in the bulk atmosphere, the influence of atmospheric CO_2 becomes noticeable, bending the $\delta^{13}\text{C}_{\text{CaCO}_3}^i$ curve towards the equilibrium value.

2) Carbonate solutions formed under lower partial pressures of CO_2 (open or closed system) precipitate CaCO_3 more slowly and with higher $\delta^{13}\text{C}_{\text{CaCO}_3}^i$ values than the solutions formed under higher P_{CO_2} values.

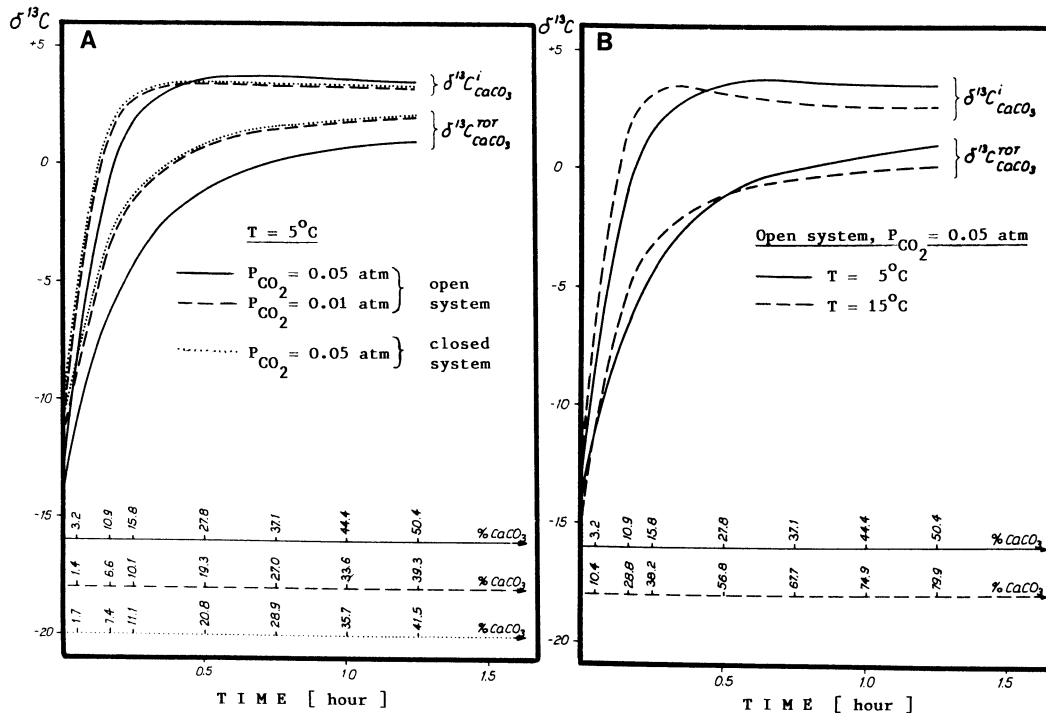


Fig 4A. Changes in $\delta^{13}\text{C}$ values of individual portions ($\delta^{13}\text{C}_{\text{CaCO}_3}^i$) and total precipitated ($\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$) calcite as a function of time elapsed since the beginning of the precipitation process, under different conditions of carbonate solution formation. Percentage of the CaCO_3 precipitated from the beginning of the process is also indicated. B. Temperature influence on $\delta^{13}\text{C}_{\text{CaCO}_3}^i$ and $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$ values during precipitation.

3) The influence of the temperature on the precipitation rate of CaCO_3 is noticeable. The change from 5–15°C results in an increased precipitation rate by a factor of 2–3, depending on the actual stage of the precipitation process. This is due to the temperature dependence of the rate constants k_{-1} and k_{-2} .

Influence of $\delta^{13}\text{C}$ of Soil CO_2

In order to examine the influence of different $\delta^{13}\text{C}$ values of soil CO_2 on isotopic composition of precipitating calcite, the necessary numerical calculation was made according to the procedure described previously. The values of soil CO_2 in the order of –30, –20 and –16‰ vs PDB was assumed.

In general, the time dependence of $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$ and $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{i}}$ of the precipitating calcite is similar to that presented on Figure 4. However, starting points of the $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{i}}$ and $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$ curves are shifted with respect to analogous points on Figure 4 calculated per $\delta^{13}\text{C}$ of the soil CO_2 equal to –25‰. For extreme values of $\delta^{13}\text{C}$ of the soil CO_2 , *ie.*, –30 and –16‰, the $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{i}}$ in open-system conditions reaches the following values respectively:

–22.0 and –6.3‰ for $P_{\text{CO}_2} = 0.05$ atm at 5°C

–17.5 and –5.4‰ for $P_{\text{CO}_2} = 0.01$ atm at 5°C

–21.1 and –7.1‰ for $P_{\text{CO}_2} = 0.05$ atm at 15°C

For the closed system with $P_{\text{CO}_2} = 0.05$ atm at $t = 5^\circ\text{C}$, the corresponding values are –13.8‰ and –6.6‰. Both $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$ and $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{i}}$ approach equilibrium with atm CO_2 as the rate of precipitation approaches zero. The higher the $\delta^{13}\text{C}$ of soil CO_2 , the more rapidly $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{TOT}}$ and $\delta^{13}\text{C}_{\text{CaCO}_3}^{\text{i}}$ increase at the initial stages of the precipitation process.

CONCLUDING REMARKS

Our model can be regarded as an up-to-date approach towards better understanding of the physical and chemical parameters controlling the isotopic composition of carbonate deposits formed in a cave environment. In numerical calculations, the most recent values of the equilibrium and kinetic chemical reaction constants were used together with the widely accepted values of carbon isotope fractionation factors. Supersaturation of the solution as a prerequisite for the initiation of precipitation was introduced in accordance with recent experimental data. The precipitation rates of CaCO_3 obtained in the model (10^{-7} – 10^{-8} mol/L s) are in satisfactory agreement with the values quoted in literature (Dreybrodt 1985).

For a temperature close to +5°C, which is typical for most mid-latitude caves, the model predicts values of $\delta^{13}\text{C}$ of precipitated calcite in the range of ca –16 to +3‰, depending on particular conditions of the precipitation process. The majority of $\delta^{13}\text{C}$ values measured in speleothems are in the range of –11 to –5‰. This, with model predictions, suggests that only several percent of the total carbon in the solution is precipitated. It is worth noting that 100% yield of the precipitation would lead to yearly growth layers of speleothems up to two orders of magnitude larger than observed in nature.

Although the agreement between measured and calculated $\delta^{13}\text{C}$ values of speleothems argues for the validity of the model, we should not forget the simplified assumptions on isotopic equilibrium between HCO_3^- and $\text{CO}_{2\text{w}}$ reservoirs in solution, the absence of backward reaction $\text{CO}_{2\text{w}} \rightarrow \text{HCO}_3^-$ and of concentration gradients for H^+ , HCO_3^- and CO_3^{2-} ions in solution.

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