Abstract—Criteria for finding asteroid families (Zappala et al. 1995) are applied to a large (205,770 member) data set of asteroid orbital elements. The cases of the Eunomia and Themis families are considered as examples. This is combined with the cratering criteria for catastrophic disruption of small bodies in the solar system (Leliwa-Kopystynski et al. 2008). We find that the Eunomia parent body itself was not catastrophically disrupted in the family-generating impact event; after impact, the current body contains as much as 70% of its primordial mass. However, by contrast with Eunomia, the present mass of 24 Themis is only about 21% of that of its primordial body. Limits are placed on the sizes of the impactors in both examples, and for the case of Eunomia, the radius of the just sub-critical crater (which may be present on 15 Eunomia) is predicted as <58 km.

INTRODUCTION

Asteroid families have long been identified in the asteroid belt (e.g., see Cellino et al. 2009 for a recent review). Here we consider the impact events that created the families. There are two extreme possibilities: a crater forming event or a catastrophic disruption event. In the former case, the current parent body of the family is essentially the pre-impact body albeit with the addition of an impact crater. In the latter case, the pre-impact body is disrupted into fragments, and after any re-accumulation as a rubble pile, the current titular head of the family contains less than half of the pre-impact parent mass. The key parameter involved is the impact energy density Q. If it exceeds a critical value (Q*), the body would break apart, and this is commonly defined as when the mass of the largest single fragment after impact is less than half of the original target mass (see Ryan [2000] for a review of asteroid fragmentation). In Fig. 1, we illustrate the full range of outcomes possible from an impact. In Fig. 1b, a large but sub-critical crater is shown to have formed as a result of an impact where Q < Q*. In Fig. 1c, Q > Q* and multiple fragments result, some quite sizable but none with mass > ½ of the original mass. In Fig 1d, Q >> Q* and as as result the parent body is broken into many small fragments, with no sizable fragments found.

In this paper, we first identify 2 families of asteroids. These are various individual asteroids that share common orbital characteristics suggesting that they have a common origin in time, i.e., they are fragments of a parent body and are assumed to have arisen from an impact event on that body. We then determine if the impact was sub-critical or over-critical. Finally, by reference to separate criteria, we place limits on the size of possible craters formed in the impacts. In this work, we use the Eunomia and Themis families as examples.

OBSERVATIONAL FOUNDATIONS

This work is based on three sets of observations. They are totally independent and different from each other. They are related to 1) the proper elements of the asteroids orbits, 2) the relation that joins the size, albedo, and magnitude for a given asteroid, and 3) the impact phenomena. We describe each of these three contributions separately as follows.

1) Identification of the asteroid families can in principle be performed by several different methods and has been widely performed for many years now (e.g., see Zappala et al. 1995). For any of them, the proper elements a, e, i of as many as possible asteroids should be known. Here we use an updated data set for 205,770 asteroids (originally from Knezevic and Milani 2006). We applied the hierarchical clustering method (HCM) (Zappala et al. 1995) to identify families.

One of the HCM approaches to determine family members is the cut-off velocity which describes separate families (see Zappala [1995] for a discussion of this parameter). Here we take the method of increasing the value of this parameter until rapid growth in the numbers of
potential family members occurs, indicating contamination of the selected sample by the broad background of asteroids. We find a cut-off velocity of 90 m s\(^{-1}\) as the limit for determining family members.

2) To estimate the total mass of the family, the sizes and the densities of asteroids forming a family are necessary. In this work, we calculate the diameter \(2R\) of an asteroid versus its absolute magnitude \(H\) and its albedo \(p_v\) according to the formula of Fowler and Chillemi (1992):

\[
2R = 1329 \times 10^{H/5} p_v^{-1/2} [\text{km}].
\]

We assumed that all asteroids forming a family under consideration have the same albedo as the parent body. In the case of 15 Eunomia (S type asteroid), we take \(p_v = 0.21\) (Lazzaro et al. 1999), and for 24 Themis (C type asteroid) we use \(p_v = 0.067\) (taken from the JPL Small Bodies Database at http://ssd.jpl.nasa.gov/sbdb.cgi#top). Therefore the formula for the asteroid volume \(V\) versus its absolute magnitude is

\[
V = 1.23 \times 10^9 \times 10^{-3H/5} p_v^{-3/2} [\text{km}^3].
\]

When applied to the members of an asteroid family, Equation 2 enables us to obtain a cumulative volume distribution for the family. One possible complication is that in the case of the Eunomia family, there have been reports that there are C-type (and even M-type) members of the family, although this is discounted by Lazzaro et al. (1999) who classify them as interlopers in the family. In addition, it is suggested that 15 Eunomia itself has albedo variations over its surface (e.g., Reed et al. 1997).

3) On the basis of observational data (see Table 1) as well as after laboratory studies, the criteria for catastrophic disruption of icy and rocky targets have been found as follows (Leliwa-Kopytyni et al. 2008):

\[
D_{c,\text{icy}} = 1.2R, \quad D_{c,\text{rocky}} = 1.6R
\]

Here \(R\) is the radius of the body and \(D\) is the crater diameter. The subscript \(c\) denotes critical, and the impact leading to the formation of a crater with a diameter larger than \(D_c\) leads to a breakup of the target. We believe that the data collected in Table 1 are all the available data concerning observed large craters on small bodies in the solar system, with the exception of the large crater on asteroid Stein recently observed by the Rosetta spacecraft (which, however, is in agreement with the limit in Equation 3; see Burchell and Leliwa-Kopytyni 2009).
The Eunomia family, as identified here, has more than 4500 members. The sizes (diameters and volumes) were estimated by means of Equations 1 and 2, based on the assumption of equal density, porosity, and albedo for all bodies in the family. The largest members have diameters $R = 264$ km (present-day Eunomia), 116 km, 105 km, 88 km, 80 km, 73 km, and so on.

Two scenarios can be considered for the post impact events:
1. the primordial body of the family may have been disrupted and undergone significant mass loss (with some re-accumulation of fragments into a rubble pile structure), or,
2. the primordial body has preserved most of its primordial mass with the addition of a large crater.

These possibilities can be considered against the criteria (mass of the largest fragment)/(mass of the target) = 0.5 (this is taken by convention as the appropriate criteria). Here we found that the total volume of the family is $1.38 \times 10^7$ km$^3$ (equivalent to a diameter of 297 km). The volume of 15 Eunomia itself is $9.7 \times 10^6$ km$^3$, and of the other fragments $3.4 \times 10^6$ km$^3$, in total. Thus the impact on the parent body of the Eunomia family led to the loss of 30% of its volume and mass. This leads to the conclusion that a large cratering, rather than disruption, event led to the formation of the Eunomia family.

The Themis family has about 9000 members. The largest of them have diameters $R = 214$ km (24 Themis), 155 km, 148 km, 141 km, 135 km, etc.. For the Themis family, the volume of the second in size fragment is equal to as much as 38% that of 24 Themis itself (compared to 8.5% in the case of the Eunomia family). The total volume of the Themis family is estimated here as $2.40 \times 10^7$ km$^3$ (equivalent to a diameter of 358 km). The volume of 24 Themis is only $5.1 \times 10^6$ km$^3$, i.e., only 21% of the volume of the family. This meets the commonly used criteria for a catastrophic disruption event (i.e., loss of more than 50% of the mass from the parent body).

See Table 2 for more complete data related to both families considered here.

## DISCUSSION

The size distribution of fragments after impact events has long been studied in the laboratory. For example, Fujiwara et al. (1977) and Nakamura and Fujiwara (1991) reported on an extensive set of laboratory experiments spanning the cratering to catastrophic disruption regimes. More experimental data is summarized in Fujiwara et al. (1989) and Ryan (2000) and references therein. It has been suggested, for example by Tanga et al. (1999), and more recently by Durda et al. (2007) that the size-frequency distributions (SFDs) of fragments of asteroid bodies after an impact event can also be used to determine if the impact was sub- or super-critical, i.e.,...
Table 2. Populations and some other results for the two families considered. $R_{m,m}$ is the radius of the main member, $V_{m,m}$ is the volume of the main member, $V_i$ is the volume of the whole population and is therefore the volume of the primordial target, $R_i$ is the radius of the primordial target, and $D_{c,t}$ is the diameter of the largest crater that could be formed on the primordial target without shattering it.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>15 Eunomia</th>
<th>24 Themis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>4500</td>
<td>9000</td>
</tr>
<tr>
<td>$2R_{m,m}$ (km)</td>
<td>264</td>
<td>214</td>
</tr>
<tr>
<td>$V_{m,m}$ ($10^6$ km$^3$)</td>
<td>9.6</td>
<td>5.0</td>
</tr>
<tr>
<td>$V_i$ ($10^6$ km$^3$)</td>
<td>13.8</td>
<td>24.0</td>
</tr>
<tr>
<td>$2R_i$ (km)</td>
<td>297</td>
<td>358</td>
</tr>
<tr>
<td>$V_{m,m}/V_i$</td>
<td>0.70</td>
<td>0.21</td>
</tr>
<tr>
<td>$D_{c,t} = 1.6R_i$ (km)</td>
<td>238</td>
<td>286</td>
</tr>
<tr>
<td>Consequence of the impact event on primordial target</td>
<td>Large crater formation</td>
<td>Catastrophic disruption</td>
</tr>
</tbody>
</table>

...and it can be seen that the Eunomia family members found here show the rapid initial drop off in fragment size (i.e., concave shape) that characterizes the crating type of event, whereas the Themis family members show a convex shape in Fig. 3 and are thus more compatible with a catastrophic disruption event. By contrast, based upon modelling that fits the observed size distribution of the family members, Durda et al. (2007) suggested both families were produced by catastrophic disruption. In the case of Eunomia, they found a diameter for the putative parent of 292 km (compared to our estimate of 297 km and 284 km by Tanga et al. 1999). However, for Themis they suggested a pre-impact diameter of 451 km, compared to our estimate of 358 km and the value of 369 km predicted by Tanga et al. (1999). It is difficult to reconcile these divergent results for Themis. In our case, the results come from a direct measurement of the family members, but uncertainties can arise from the choice of albedo or from incompleteness of the data set. Concerning the albedo, there is some evidence that for the Themis family the albedo varies with fragment size (see Florczak et al. 1999). Regarding the incompleteness of the data set at small sizes, it can be seen from Fig. 2 that the data for Themis is incomplete (i.e., has a cut-off) at a larger size for the Themis family than for Eunomia. If there are members missing at small sizes, this will not greatly influence the total family mass only if either the cut-off is at a sufficiently small size, or most of the mass is found in the larger family members (i.e., Q is not >> than Q$^*$).

Durda et al. (2007) suggest that methods based on counting asteroids (as here) underestimate the pre-impact mass of the parent body by ~90% (and thus a factor of 2 in diameter). They considered the case of the Hoffmeister family, which they describe as arising from a super-catastrophic disruption event. However, we note that for the Eunomia family, the counting method used here produces an estimate of the parent body size similar to that of Durda et al. (2007). We suggest this could be because either the set of asteroid orbits used in this paper is complete at relatively small sizes, or because the impact is here considered to be sub-critical and this is producing a smaller fraction of the post-impact mass in small fragments (and reconstruction of such families is thus inherently less sensitive to the lack of completeness at small sizes). However, before asserting a general rule that counting methods work for sub-critical but not super-catastrophic impacts, we note that 24 Themis is a C-type asteroid. Durda et al. (2007) specifically caution that their model may not apply to C-type asteroids due to their porosity, which may influence the impact process (e.g., see Love et al. 1993; Housen and Holsapple 2003; Burchell et al. 2005 for examples of how porosity alters the outcome of impact events at laboratory scales).
We next consider the impact events themselves. Durda et al. (2007), suggest that, based on their modelling, the predicted $Q^*$ values of Benz and Asphaug (1999) are appropriate for asteroid families. Impact speeds in the asteroid belt have been estimated previously as, for example, according to Farinella and Davis (1992) $v_{\text{imp}} = (5.81 \pm 1.88) \text{ km s}^{-1}$; Bottke et al. (1994) have found 5.3 km s$^{-1}$ and Wlodarczyk (1995) obtained 5.3 km s$^{-1}$ (by considering the statistics of close encounters of Vesta with the first 2000 numbered minor planets in the years 1770–2170). In this work, given the range of the predictions combined with the spread given by Farinella and Davis (1992), for convenience we assume $v_{\text{imp}} = 5 \text{ km s}^{-1}$. This value also has the merit that it is one of the speeds used in the modeling of Benz and Asphaug (1999). Accordingly, we use the results of Benz and Asphaug (1999) for impacts on basalt at 5 km s$^{-1}$ and random angles of incidence, and we obtain that $Q^*$ is $7.65 \times 10^5 \text{ J kg}^{-3}$ for the Eunomia family parent body, and $9.87 \times 10^5 \text{ J kg}^{-3}$ for the Themis family parent body.

Having obtained $Q^*$ we need to know the parent mass to obtain the necessary impact energy. Given our calculated volumes for the parent bodies, we can obtain a mass using the appropriate density. The issue of the difficulty in determining accurate asteroid densities is discussed in Hilton (2002). For 15 Eunomia, Michalak (2001) finds a density of $3.2 \pm 1.7 \text{ g cm}^{-3}$, although we note that the uncertainty is about 50%. More recent estimates suggest both higher (Vitagliano and Stoss 2006) and lower (Kochetova 2004) masses for 15 Eunomia and thus different densities (30% higher and 26% lower, respectively). For the Themis family, we have a choice. Generally, C-type asteroids are held to have low densities due to their porosity and values of ~1.8 g cm$^{-3}$ are often given. However, Michalak (2001) finds that an average density for 5 C-type asteroids is $2.9 \pm 0.2 \text{ g cm}^{-3}$. In the absence of well-constrained precise measurements for either family, we assume a common density of 2.7 g cm$^{-3}$ for both families and the associated impactors. While clearly an assumption, it is compatible with known measurements and is also the value used in the modeling of Benz and Asphaug (1999).

We thus obtain that the radius of the impactor needed to just cause a catastrophic disruption on the Eunomia parent
body was 58 km, whereas that for the Themis parent body was 77 km. Thus to be a sub-critical cratering event, the Eunomia family was created by impact of an object of less than 58 km in radius, whereas the Themis family creating event required an impact by an object of a radius greater than 77 km. The mass ratio (projectile/target) for the Eunomia family creating event was $<0.061$, whereas that for the Themis family was $>0.079$.

The issue of the size of the impact crater on 15 Eunomia then arises. Here we use Equation 3 to set a limit. If the impact had just been at the critical threshold for disruption, then given the predicted size of the Eunomia family parent, the largest crater it could sustain would have had a diameter of 238 km, thus the expected impact crater on 15 Eunomia is $<238$ km in diameter. However, given that the impact was only just sub-critical it will not be significantly smaller than this and should still be apparent on 15 Eunomia. We next compare our prediction for the impactor radius to that obtained from scaling laws in the gravity regime (e.g., Schmidt and Housen 1987) as discussed for the impact crater Stickney on Phobos by Asphaug and Melosh (1993), where the impactor radius ($r_i$) is given by:

$$r_i = 0.41 \left( \frac{D_c}{g} \right)^{1.28} v^{-0.56} .$$

We thus find that a crater on the Eunomia parent with diameter 238 km requires an impactor of radius 13 km (if traveling at 5 km s$^{-1}$). However, this is for normal incidence. In cratering efficiency studies at non-normal incidence, it is common to replace the impact speed with its vertical component (e.g., see discussion in Melosh 1989). If this is done in Equation 4, the size of the impactor increases slightly to 16 km. Equivalently, in the calculations using $Q^*$, the value for $Q^*$ can be taken as that at normal incidence (rather than that averaged over all angles) and Benz and Asphaug (1999), show that this is approximately a factor of two smaller than the average value. Thus at normal incidence, the $Q^*$ method predicts an impactor with a radius of 46 km. Thus, the two predictions converge slightly, but are still a factor of approximately three apart.

**CONCLUSIONS**

The statistical method of identifying asteroid family members, combined with a large data set of known asteroids, permits a ready identification of both asteroid families and their members. If the list of family members is (relatively) complete, this in turn allows the pre-family mass of the parent to be estimated. Based on the criteria traditionally used to define disruption events, the nature of the impact event, large crater or catastrophic disruption, is then determined. Two examples are given here, Eunomia is an example of the former (cratering), Themis an example of the latter (catastrophic disruption). In the case of Eunomia, the post-impact body retained 70% of its original mass, whereas Themis only retained 21%.

This difference in nature between the two families is important. As often pointed out (e.g., Cellino et al. 2009), asteroid families are good targets for space missions, as the individual members may reveal the internal structure of the parent (depending on where inside the parent the fragment originated). However, if a family originated from a cratering, rather than disruption, event this may not be quite so revealing. In the case of a just sub-critical impact, it may be that the target body has been disrupted to an extent. If it was a differentiated body for example, then such an impact may have stripped away the mantle exposing the core, and some re-accumulation of ejecta under gravity may cover up some of the exposed core. This would lead to variations in albedo across the body as reported by Reed et al. (1997) for 15 Eunomia.

To obtain limits on the size of the impactor which caused these events, the mean collisional speed in the asteroid belt was assumed and combined with the modelling of Benz and Asphaug (1999). In the case of the Eunomia family, an upper limit was obtained, for Themis it is a lower limit. This is however based on assuming that the parent bodies were themselves monolithic, i.e., not rubble pile re-assemblies of earlier impact events. In addition, a prediction is made for the upper size of the crater expected on 15 Eunomia. This was then used to obtain a second, separate estimate of the size of the necessary impactor using scaling laws (rather than on modelling of $Q^*$) and this was found to be smaller than our model based estimate by a factor of approximately three.

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The impact origin of eunomia and themis families


