

## Chondrule collisions in shock waves

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Abstract—Detailed numerical models have shown that solar nebula shock waves would be able to thermally process chondrules in a way that is consistent with experimental constraints. However, it has recently been argued that the high relative velocities that would be generated between chondrules of different sizes immediately behind the shock front would lead to energetic collisions that would destroy the chondrules as they were processed rather than preserving them for incorporation into meteorite parent bodies. Here the outcome of these collisions is quantitatively explored using a simple analytic expression for the viscous dissipation of collisional energy in a liquid layer. It is shown that molten chondrules can survive collisions at velocities as high as a few hundred meters per second. It is also shown that the thermal evolution of chondrules in a given shock wave varies with chondrule size, which may allow chondrules of different textures to form in a given shock wave. While experiments are needed to further constrain the parameters used in this work, these calculations show that the expected outcomes from collisions behind shock waves are consistent with what is observed in meteorites.

# INTRODUCTION

Chondrules, the millimeter-size, igneous spherules that dominate the textures of primitive meteorites, have long been recognized to be the result of transient heating events that took place during the first few million years after the collapse of the molecular cloud from which our solar system formed (for reviews, see Hewins 1997; Jones et al 2000; Connolly and Desch 2004). Identifying the source of these heating events has proven to be difficult, as the source must process the chondrules in ways that explain a long list of petrologic and chemical constraints, while also fitting within the context of models for how our solar system formed and evolved. Understanding chondrule formation is interdisciplinary effort, requiring detailed comparisons between model predictions and the meteoritic record.

Recently, shock waves within the solar nebula have become the leading candidates for the sources of these transient heating events (Boss 2004). Numerical models have shown that silicate particles that were overrun by nebular shock waves would be heated and cooled on time scales consistent with what has been inferred for chondrules (Iida et al. 2001; Desch and Connolly 2002; Ciesla and Hood 2002). These shock waves could be created by gravitational

instabilities within the solar nebula (e.g., Boss 2002) or by the gravitational interaction of Jupiter with the surrounding protoplanetary disk (Bryden et al. 1999; Boley et al. 2005). It is also possible that X-ray flares from the young sun produced shock waves in the upper layers of the solar nebula (Nakamoto et al. 2005).

The way that the shock wave model for chondrule formation works is as follows: the shock front represents a discontinuity between the cool, low-density, upstream gas that is moving rapidly toward the shock front and the hot, dense, post-shock gas that moves slowly away from the shock front. Solid particles (chondrule precursors) are assumed to be present in the nebula and initially are in kinetic and thermal equilibrium with the pre-shock gas (no relative velocity exists and the temperatures are equal). As the nebular gas passes through the shock front, its properties (temperature, density, velocity) change in a way that is predicted by the Rankine-Hugoniot relations (Landau and Lifschitz 1987). The solid particles that are entrained in the gas pass through the shock front unaffected, retaining their pre-shock temperatures and velocities. This results in a state of disequilibrium, as lowtemperature, rapidly moving solids are surrounded by hightemperature, slowly moving gas. Energy and momentum are exchanged between the solids and gas resulting in the solids

being heated very rapidly as they lose their velocities with respect to the gas. As the chondrules lose their velocity with respect to the gas, that rate of heat exchange decreases in intensity, and the particles begin to cool as they radiate away energy.

While numerical models have shown that the thermal evolution of chondrules can be explained by processing in shock waves, these efforts open the doors to more questions. If shock waves were responsible for the formation of chondrules, the proper peak temperatures and cooling rates that the chondrules would experience require the proper combination of gas density, shock velocity, and solid concentration (Iida et al. 2001; Desch and Connolly 2002; Ciesla and Hood 2002). It is unclear how the solar nebula and the shock-wave-generating mechanism (whatever it may be) worked either separately or in concert so that these conditions were met, or why there is no record of objects that were processed in conditions that did not fall in the "chondrule formation" range. In addition, chondrules exhibit a number of properties that must be explained beyond just their thermal evolution. For example, while chondrules were processed at high temperatures, conditions were such that isotopic fractionation was kept to a minimum (Alexander 2004). In order for a chondrule formation theory to be complete, it must explain the bulk properties of chondrules and fit within our context of how the solar nebula evolved (Ciesla 2005).

Some studies have begun to explore these issues in the context of the shock wave model. For example, it has been argued that chondrule formation by shock waves may be able to explain the observed number of compound chondrules in chondritic meteorites. Compound chondrules are two or more chondrules fused together, and some, if not all, of these objects are expected to form from the collisions of chondrules while at least one was molten. Because shock waves will spatially concentrate particles, the collisional frequency of chondrules will be increased over what it would be in an undisturbed nebula. Several previous studies have concluded that the proper number of compound chondrules can be formed, provided that the solids are concentrated in the preshocked nebula at levels near 30-50 times what they would be under canonical conditions (Desch and Connolly 2002; Ciesla and Hood 2002; Ciesla et al. 2004a).

The above studies only considered the relative velocities between chondrules due to turbulence or the random motions that were assumed to be present within the gas. Within the context of the shock wave models, relative velocities between the chondrules may also develop due to the deceleration of the objects by the gas. The rate at which a particle decelerates depends on its size, and thus different sized particles would decelerate at different rates. Because of the large velocity difference between the initial velocity and final velocity of chondrules in the shock wave model (~5 km/s or greater), the velocity difference between particles of different sizes can be quite large. This led Nakmoto and Miura (2004) and Uesugi et al. (2005) to conclude that shock waves would produce

collisions that destroyed chondrules rather than allowing them to survive and be incorporated into meteorite parent bodies.

In this paper, the evolution of different sized particles as they are processed in a given shock wave is investigated. In the Chondrule Dynamics in Shock Waves section, the dynamics of chondrules in a shock wave and the relative velocities that would develop between particles of different sizes are described. In the Chondrule Collisions section, the outcomes of chondrule collisions that occur over the range of relative velocities expected in shock waves are discussed. Other issues that must be considered when trying to determine the outcome of collisions in the shock wave model are presented in the Other Factors in Chondrule Collisions section. In the Note on the Textures of Compound Chondrules Components section, an overview of how the thermal evolution of a chondrule in a shock wave varies with size is presented and the observations of the textures observed in compound chondrules are discussed. The implications for these results are discussed in the Discussion section.

## CHONDRULE DYNAMICS IN SHOCK WAVES

As a chondrule-sized particle moves through a gas in the solar nebula, it loses its relative velocity with respect to the gas in a time given by the stopping time,  $t_s$ , given by:

$$t_{\rm s} = \rho r / \rho_{\rm o} c \tag{1}$$

where  $\rho$  is the mass density of the particle, r is the particle radius,  $\rho_g$  is the density of the nebular gas, and c is the local speed of sound in the gas. This expression is valid for particles whose radius is less than the mean free path of the gas molecules, which is a condition that is easily met by chondrules in nebular gas. As can be seen from this relation, the stopping time is proportional to the chondrule radius, meaning smaller particles will lose their velocities with respect to the gas over a shorter period of time than larger particles. It is this effect that would cause particles of different sizes to develop velocities with respect to one another as they are processed by a shock wave.

More explicitly, as a chondrule enters a shock wave, its velocity will change as a function of distance as described by:

$$mv_{c}\frac{dv_{c}}{dx} = -C_{D}\frac{\pi}{2}r^{2}\rho_{g}(v_{c} - v_{g})|v_{c} - v_{g}|$$
 (2)

where  $v_c$  and  $v_g$  are the velocities of the chondrule and gas with respect to the shock front and  $C_D$  is the drag coefficient which is a function of the temperatures of the particle and the gas and the velocity of the particle with respect to the gas (Ciesla and Hood 2002). It is assumed here that the gas velocity is a constant in the above expression. Figure 1 shows how the total velocities of three chondrules of different size  $(r=0.2,\ 0.3,\ \text{and}\ 0.5\ \text{mm})$  decrease as they enter a shock wave. The gas is assumed to initially be at 400 K and a density of  $4\times 10^{-10}\ \text{g/cm}^3$  with the shock moving through at a

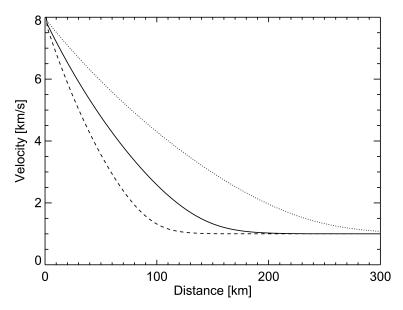


Fig 1. The velocities of chondrules with respect to the shock front as a function of distance after entering the shock. Plotted are chondrules with radii of 0.5 (solid), 0.3 (dotted), and 0.2 mm (dashed). The pre-shock gas is assumed to be at a density of  $4 \times 10^{-10}$  g/cm<sup>3</sup> and an initial temperature of 400 K. The shock velocity is 8 km/s with respect to the undisturbed pre-shock gas.

velocity of 8 km/s. These values are typical of the conditions expected in shock waves that would form chondrules in the central scale height of the solar nebula (Desch and Connolly 2002; Ciesla and Hood 2002; Ciesla et al. 2004b). The gas immediately behind the shock increases in density to  $\sim 2.4 \times$  $10^{-9}$  g/cm<sup>3</sup> and recedes from the shock front at ~1 km/s. The gas properties change as described in Ciesla et al. (2004b) with negligible effects from the interaction with the solids due to the low assumed number density of solids. In addition, the radiation field was set to be constant at the equivalent of a blackbody radiating at 1500 K to represent the ambient field due to the hot chondrules immediately behind the shock front. The material properties of the chondrules were chosen to be equal (density of 3.3 g/cm<sup>3</sup>, wavelength averaged emissivity of 0.9, and heat capacity of 107 erg/g K), ensuring that all differences in their dynamical and thermal evolution were due to particle size alone. (It should be noted that density differences could also lead to relative velocities developing between chondrules. Here we focus on the differences in chondrule sizes because they range over an order of magnitude, while chondrule densities likely only vary by no more than a factor of 2.) The particles all eventually come to rest with respect to the gas, receding from the shock front at ~1 km/s. Because of the different sizes of the particles, they reach this kinetic equilibrium with the gas over different distances. This causes relative velocities to arise between the particles, and the magnitudes of these relative velocities are shown in Fig. 2.

The relative velocities between the different sized particles are on the order of 1 km/s for the first few hundred kilometers behind the shock front. If the shocks were to take place in the upper layers of the disk, such as those caused by

X-ray flares (Nakamoto et al. 2005), the required shock velocity would be much greater to melt chondrules due to the lower gas density, leading to larger velocity differentials. The velocity differential between two chondrules is greater when there is a larger difference in size, and the distance over which a relative velocity exists will be greater for larger particles due to the longer stopping times. These large relative velocities could lead to frequent and energetic collisions between chondrules while in their molten state. If these collisions are destructive, then upper limits can be placed on the amount of solids present in a chondrule-forming event (Nakamoto and Miura 2004). If chondrules were able to survive these collisions, then signatures of these energetic collisions may be preserved in the meteoritic record.

#### CHONDRULE COLLISIONS

As has been shown, the relative velocities between particles in a shock wave can be large, which would result in very energetic collisions. Kring (1991), using the surface tension of silicate melts, argued that compound chondrule forming collisions must have occurred at velocities less than ~100 cm/s, otherwise the collisions would result in the destruction of the molten particles rather than their adhesion. This estimate can be derived from looking at the Weber number (*We*) of a liquid sphere, or the ratio of the impact energy to the surface energy of the sphere, defined algebraically as:

$$We = 2\rho v^2 r / \gamma \tag{3}$$

where v is the velocity with which one sphere encounters the other and  $\gamma$  is the surface tension (surface energy) of the

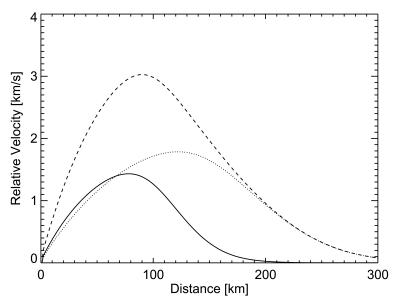


Fig. 2. Plotted are the relative velocities between the different sized particles in Fig. 1 that arise due to the different rates they decelerate behind the shock front. The solid line represents the relative velocity between the 0.2 and 0.5 mm chondrules. The dotted line is the relative velocity between the 0.2 and 0.5 mm chondrules. These relative velocities could lead to energetic collisions between chondrules of different sizes.

liquid. For values of *We* below some critical value ( $\sim$ 10), the impact transfers a small amount of energy to the liquid in comparison to the surface energy, and thus the liquid sphere remains intact. (The actual value of the critical Weber number depends on the situation being considered as will be discussed below.) For large values of *We*, the impact is much more energetic than the surface energy of the liquid, and the liquid sphere is expected to be disrupted. Using typical values for chondrules,  $\rho = 3.3 \text{ g/cm}^3$ , r = 0.3 mm, and  $\gamma = 400 \text{ dyne/cm}$  (0.4 N/m) (Susa and Nakamoto 2002), we find that collisional velocities greater than 140 cm/s would result in the disruption of the chondrules, in agreement with Kring (1991).

Ignored in this consideration is the fact that the chondrule melt will be viscous, which may allow some of the energy in the collision to be dissipated by displacement of the melt, and therefore allow more energetic collisions to occur without leading to disruption. A measure of the importance of liquid viscosity relative to surface energy is the Ohnesorge number (*Oh*), given by:

$$Oh = \eta/(2\rho r\gamma)^{1/2} \tag{4}$$

where  $\eta$  is the dynamic viscosity of the liquid. If Oh > 0.1, then viscous effects start to become important (Nomura et al. 2001). Using the values from above, this means that if the chondrule viscosity is greater than 1 poise (0.1 Pa s), which is roughly the lower limit expected for completely molten chondrules at their peak temperatures (Uesugi et al. 2005; Rubin and Wasson 2005), viscous effects must be considered when looking at the outcome of chondrule collisions.

The net effect of the viscosity is to increase the critical Weber number needed before disruption of the liquid sphere occurs. In looking at the movement of liquid spheres through a fluid, the critical value of the Weber number that determines when the ram pressure of the fluid exceeds the surface tension of the liquid has the form (Elkins-Tanton et al. 2003):

$$We_{\text{crit}} = 12(1 + 1.077Oh^{1.6}) \tag{5}$$

This formula could be used to investigate the ability of a molten chondrule to survive the high-speed flow that it would experience as it moves through the gas in a shock wave. A similar investigation was done by Susa and Nakamoto (2002) who, by assuming that  $We_{\rm crit} = 6$ , found that the balance between ram pressure and surface tension of a chondrule easily allowed particles with r < 1 mm to survive formation in a shock wave. The above equation suggests that the critical radius may be two times greater if viscosity is negligible, and even larger if the chondrule has a viscosity high enough to make Oh > 1.

In the case of a droplet colliding with a flat surface, the critical Weber number is given by the expression (Walzel 1980):

$$We_{\rm crit} = 7.9 \times 10^{10} Oh^{2.8} \tag{6}$$

which leads to significantly higher values of *We*<sub>crit</sub>. These expressions demonstrate that both the viscosity of the droplet and the geometry or properties of the impacting "object" play major roles in determining the outcome of a collision. Neither of the above formulae is appropriate for the case of a liquid sphere colliding with another liquid sphere; however, they demonstrate that considering the viscosity of the sphere will increase the velocity above which catastrophic disruption will occur.

While an expression for the critical Weber number appropriate for this collisional geometry is missing and must be determined by experiments, the deceleration that occurs when two semi-molten chondrules collide can be examined. It is assumed that the chondrules have solid cores of radii  $r_1$  and  $r_2$ , and that they are mantled by viscous layers whose thicknesses are small compared to the radii of the host chondrules. (In reality, chondrules, as they were being thermally processed, were mixtures of melt and crystals whose structures were not exactly solids surrounded by liquid as described here. However, the focus here is on the effects of the viscosity of the chondrule melt, regardless of the interior structure, thus allowing the model below to be applied.) The relative velocity, u, between two chondrules colliding head-on will decrease upon contact at a rate given by:

$$m_{\rm m}\frac{du}{dt} = -3\pi\eta r_{\rm m}^2 \frac{u}{x} \tag{7}$$

where x is the separation between the solid cores and  $r_{\rm m}$  and  $m_{\rm m}$  are the mean chondrule radius and mass given by

$$r_{\rm m} = \frac{r_1 r_2}{r_1 + r_2} \tag{8}$$

and

$$m_{\rm m} = \frac{m_1 m_2}{m_1 + m_2} \tag{9}$$

respectively (Ennis et al. 1991; Liu et al. 2000). The equation for the viscous dissipation of the collisional velocity was first derived from theory by Ennis et al. (1991) to determine the requirements for coalescence between particles with a binder layer on their surfaces. Specifically, the right side of the equation represents the force that develops as the pressure in the liquid bridge between the two particles when they come into contact as predicted by viscous lubrication theory. In deriving this equation, it was assumed that the viscous dissipation dominates the dissipation compared to any capillary effects of the liquid. As can be seen, the magnitude of the deceleration increases with increasing viscosity, meaning that more viscous melts will lead to greater energy dissipation in a given collision. This is consistent with the experimental results of Willis and Orme (2003), who found that energy dissipation in droplet collisions increases as the viscosity of the liquid is increased.

A drawback to this treatment is that it assumes that the viscosities of the surface layers on each particle are equal. Below it is shown that this was not necessarily the case when chondrules collided with one another. In this case, it is likely the lower viscosity that determines the magnitude of the deceleration.

More detailed investigations of particle impacts have been developed. For example, Liu et al. (2000) not only considered the viscous dissipation in the binder layer during a collision of the type described above, but also investigated the energy loss during deformation of the solid substrate when they came into contact. Unfortunately, this work and that of Ennis et al. (1991) were motivated to study granulation phenomena for industrial purposes, and therefore the validity of these equations have only been experimentally tested in low velocity regimes (u < 1000 cm/s), and not at the higher velocities expected to develop between chondrules behind shock waves. Because of the lack of experimental support (because the experiments have not been done), the focus here is on the simple case of dissipation in the viscous layer to investigate the first order effects that can be expected in chondrule collisions.

The above equations were used to study the outcomes of collisions between chondrules for a variety of sizes and velocities. Each chondrule is assumed to have a surface viscous layer that is 10% of its radius. The relative velocities between the two chondrules were calculated as a function of the normalized separation (the separation between their solid centers divided by the separation at the initial contact). If the solid centers came to rest with respect to one another while retaining some separation ( $x/x_0 > 0.01$ ), then the energy of the collision is assumed to have been dissipated within the viscous layer and the chondrules survive the collision.

If there is a residual velocity when the cores come into contact, then the outcome of the collision cannot easily be predicted. The cores could fracture causing the two spheres to be destroyed, or the cores could further dissipate the energy of the collision and allow the spheres to still survive. Thus the criterion for chondrule survival used here is likely conservative, as it is possible that other effects may prevent chondrules from being disrupted, rather than just the viscous dissipation in a thin layer of melt. In order to fully investigate these collisional outcomes, a more detailed model is needed that accounts for the rheology of the solids. The point of the one-dimensional investigation here is to demonstrate the effect that the viscosity of the chondrule melt would have on the outcome of collisions.

Uesugi et al. (2005) and Rubin and Wasson (2005) both estimated that chondrule viscosities would be on the order of 1 poise at temperatures near ~1900 K if the chondrule is completely molten. What is particularly important in determining the outcome of a collision is the viscosity of the chondrule melt at the time of impact, which for a particular chondrule would fluctuate rapidly because of the initial rapid cooling (>200 K/min) that occurs behind a shock wave (Ciesla and Hood 2002; Desch and Connolly 2002). Chondrule melt viscosities likely fell into a wide range of values, as they would vary greatly with composition, temperature, and degree of melting. Determining the exact temperature dependence of a multicomponent silicate melt, such as is expected for chondrules, is the subject of ongoing work. Many natural melts exhibit viscosities ranging 6 orders of magnitude over the temperature range of interest for

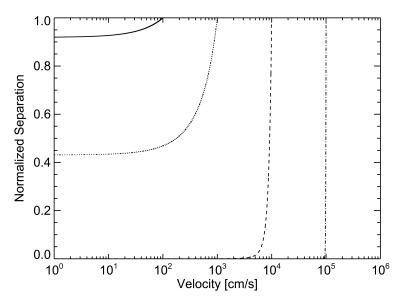


Fig. 3. Plotted are the normalized separations of the rigid components of colliding 0.5 and 0.3 mm chondrules as a function of velocity. The chondrules are assumed to have surface layers that are 10% of their respective radii and have viscosities of 100 poise. The collisions initially occur at a normalized separation of 1. During the collision, the relative velocity of the chondrules decreases as the surface layers dissipate the energy of the collision. For collisional velocities below 100 m/s, the relative velocities between the chondrules disappear before the rigid centers come into contact, and thus catastrophic disruption is avoided.

chondrule formation (Giordano and Dingwell 2003). The issue is complicated further by the fact that not all chondrules completely melt, and thus the viscosity will be determined by the composition of the liquid present rather than the bulk composition of the chondrule. For instance, albite is often found in the mesostasis of chondrules (Grossman and Brearley 2003) and has a viscosity that ranges from 100 poise at 2000 K to nearly 10<sup>5</sup> poise at 1600 K (Russell et al. 2003). While albite would not be the only melt present in the chondrule, this does demonstrate that liquids expected in a partially molten chondrule had viscosities that varied by orders of magnitude and exceeded the 1 poise estimate of other workers. Thus the viscosities of chondrule melts at the time of collisions likely ranged over many orders of magnitude.

Figures 3 and 4 show the results of collisions at different speeds between chondrules with radii of 0.5 mm and 0.3 mm at velocities of 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup> cm/s. In Fig. 3, it is assumed that the viscous layers at the surface of the chondrules had viscosities of 100 poise, while in Fig. 4 a value of 1000 poise was assumed. These viscosities are similar to those predicted for chondrules from Tieschitz for the temperature range of 1500-1900 K (Gooding and Keil 1981). The results shown in Figs. 3 and 4 demonstrate that viscous chondrule melt can easily dissipate the collisional energy between chondrules with encounter velocities as high as 100 m/s, which is two orders of magnitude greater than the upper limit estimated based on the surface tension of the melts by Kring (1991). Even greater collisional velocities are allowed when smaller chondrules collide with large ones that is, when the size ratio is less than the case considered here (the smaller mass results in less collisional energy). Lower viscosities than those used here would require lower collisional velocities for the chondrules to survive, using the conservative criteria that all impact energy must be dissipated in the surface melts of the chondrules. Also, if the collisions took place between less massive chondrules (or some combination producing lower values of  $m_{\rm m}$ ), the energy of the collision would be dissipated more easily.

Based on these calculations, chondrule destruction likely only happened immediately behind the shock front when the chondrules were near their peak temperatures (were at their lowest viscosity) and had relative velocities with respect to one another that were in excess of ~100 m/s. In addition, these collisions may only be disruptive if the chondrules are on the large end of the chondrule size range. The amount of time during which the relative velocities exceed this value is approximately 100 s in the shock wave results described here. After that time period, collisional velocities would be such that the energy of collisions would be dissipated in the melt present in each chondrule, allowing compound chondrules to form.

To quantify this, the average number of collisions that a single chondrule would experience with chondrules of another size is given by:

$$N \sim \pi (r_1 + r_2)^2 v_{\rm rel} n_{\rm c} t$$
 (10)

where  $v_{\text{rel}}$  is the relative velocity between the chondrule of radius  $r_1$  and chondrules of radius  $r_2$ ,  $n_c$  is the number density of particles with radius  $r_2$ , and t is the amount of time that the relative velocity exists. During the period immediately after the chondrules entered the shock wave,

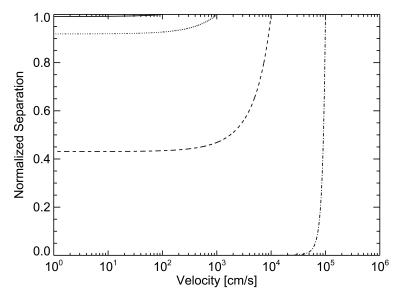


Fig. 4. Same as Fig. 3, except the surface layers are assumed to have viscosities of 1000 poise. The higher viscosity allows the collisional energy to be dissipated over shorter distances, leading to less deformation of the chondrules. Collisional velocities of 1 km/s are still problematic, but velocities greater than 100 m/s would not necessarily lead to chondrule disruption.

their relative velocities would be on the order of 1 km/s for 100 s. In the case of chondrules with  $r_1 = 0.5$  mm and  $r_2 =$ 0.3 mm, then the number density of chondrules required to give N = 1 is  $n_c = 5$  per cubic meter, with this number going up for smaller chondrules. This is the density of chondrules in the relaxation zone of the shock, where the chondrules are still moving with respect to the gas. Thus these particles will not have reached the maximum number density they would achieve when their motions relative to the gas had stopped. The number density calculated here would have to be similar to the number density that the chondrules were concentrated at prior to entering the shock. In the chondrule-forming shock waves investigated by Desch and Connolly (2002) and Ciesla and Hood (2002), the pre-shock gas was considered to be at a density near  $10^{-9}$  g/cm<sup>3</sup>. The number density calculated here would give a chondrule mass density of ~2 × 10<sup>-9</sup> g/cm<sup>3</sup>, which requires an enhancement of ~400 times that expected under canonical conditions. Such enhancements are predicted to occur in the disk, but they would occupy only small volumes of space which could make chondrule formation in such regions rare (Cuzzi et al. 2001). The same number density could be achieved at a lower enhancement if the ambient gas density was higher, but this would require smaller shock velocities to melt the chondrules, and thus smaller relative velocities would exist for shorter periods of time. Smaller chondrules could be concentrated at a higher number density without changing the mass density, but as discussed above, their lower mass would make them less likely to cause collisional disruption. Thus it is unlikely that collisions would be common immediately behind the shock front when chondrules are most susceptible to disruption.

## OTHER FACTORS IN CHONDRULE COLLISIONS

In addition to the viscous effects described above, there are other factors that must be considered when examining the collisions between chondrules behind shock waves. In the above discussion, it was assumed that shock waves would process particles over a range of sizes as they passed through the nebula. This is reasonable, since chondrules are observed to range in diameter from roughly 0.1 to 1 mm. However, in a particular meteorite type, the chondrules appear to be concentrated around a particular size, suggesting that sizesorting took place in the disk. If regions of the nebula are concentrated with particles of similar size, as is proposed to occur in turbulent concentration (Cuzzi et al. 2001), then the particles processed by shock waves will have similar aerodynamic properties (stopping times) and would not develop large relative velocities with respect to one another. If pre-shock sorting took place, then large enhancements of particles would be possible without having to worry about catastrophic disruption. More work is needed to understand if there was any size-sorting of chondrules prior to their processing.

Also, in this work and that of other authors, it is assumed that the collisions between chondrules were head-on—that is, that the relative velocity between these objects was along the line of centers. Such a situation would maximize the likelihood that chondrules would be disrupted in a collision. However, the probability of such a collision would be low. Most collisions likely were off-axis, with a finite value of the impact parameter, *b*, or distance between the parallel trajectories of the chondrules in the center-of-mass reference frame. This would result in only part of the collisional energy

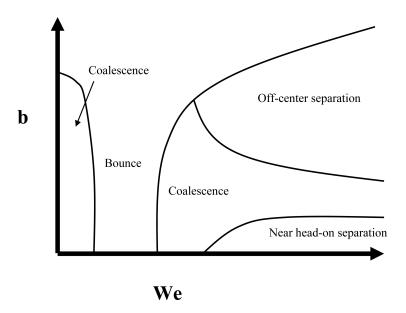


Fig. 5. A schematic diagram taken from Qian and Law (1997) demonstrating how different collisional outcomes can be expected depending on the Weber number (*We*) and impact parameter (*b*) of a given collision between liquid spheres. When coalescence occurs, the two impacting droplets remain fused at the end of the collision. When the droplets bounce, the objects do not fuse, but instead deform slightly but remain two individual spheres with no mass loss. The droplets also do not fuse in the case of separation, but do lose some mass depending on the dynamics of the collision. In many cases, the amount of mass lost is only a small fraction of the original droplets. Experiments are needed to quantify where the different boundaries are between the outcomes for molten chondrules.

being transferred to each chondrule, and thus reduce the likelihood of disruption. In fact, experimental work has shown that there are a variety of outcomes expected in collisions between liquid spheres depending on the Weber number and impact parameter, as shown in Qian and Law (1997). An example of how the collisional outcome varied with collisional parameters is shown schematically in Fig. 5.

In these experiments, three outcomes were observed: coalescence, bouncing, and separation. Coalescence resulted in the merging of two droplets, which would be analogous to the formation of compound chondrules. Bouncing resulted in the droplets colliding with one another but not merging, preserving the original droplets at the end of the collision. Finally, separation, which was observed at high values of We, resulted in some mass loss from the original droplets. However, the amount of mass lost was often a small fraction of the original droplets, still leaving two dominant droplets. If such an outcome were to occur during a collision between chondrule precursors, the two droplets may have lost mass, yet at the end of the collision still form chondrules. Thus while energetic collisions may occur in shock waves, the products of those collisions could still individually become chondrules themselves. The fact that not all collisions result sticking suggests that estimates of chondrule concentrations in a given formation event that are inferred based on compound chondrule statistics should be taken as minimum values as the compound chondrules representative of only a fraction of the collisions that took place.

# A NOTE ON THE TEXTURES OF COMPOUND CHONDRULES COMPONENTS

In addition to the energetic collisions between chondrules, Uesugi et al. (2005) argued that another problem for the shock wave model is the fact that many compound chondrules consist of components with different textures. Wasson et al. (1995) examined the textures of 80 compound chondrules and noticed that approximately 25% contained one chondrule with a porphyritic texture while the other had a non-porphyritic texture. Uesugi et al. (2005) argued that such objects could not have formed in a shock wave because chondrule textures are indicative of the thermal evolution the chondrules experienced, and thus objects processed in the same shock wave should have similar textures.

While the thermal history that chondrules experience will play a role in determining their textures (e.g., Radomsky and Hewins, 1990), there are a number of other factors that must also be considered (H. Connolly, personal communication; Lofgren 1996; see also Tables 1–3 in Desch and Connolly 2002). The texture a chondrule will develop also depends on the number of nucleation sites present in the chondrule melt, whether they survive the melting of the chondrule or are introduced through the collision of the chondrule melt with dust grains (Connolly and Hewins 1995). Porphyritic textures generally form when chondrules were raised to temperatures that did not exceed their liquidus temperatures thus preserving nucleation sites for crystal growth, whereas non-phorphyritic textures form when chondrule precursors were

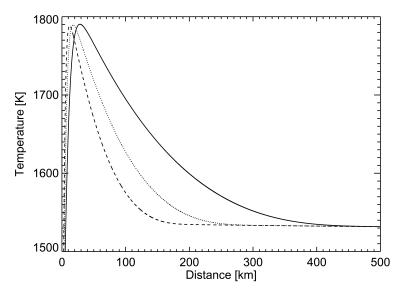


Fig. 6. Plotted are the temperatures of chondrules 0.5 (solid line), 0.3 (dotted line), and 0.2 mm (dashed line) in radius as they are processed by the shock wave described in the text.

completely melted (heated above their liquidus temperatures) and all nucleation sites were destroyed. The liquidus temperature of a chondrule depends on its composition, so two chondrules of even slightly different composition could develop different textures if processed in the same shock wave as the temperature that they are heated to may exceed the liquidus for one chondrule but not the other. The grain size distribution of the chondrule precursors may also play a role in determining their textures in a given formation event (Lofgren 1996).

In addition to the chemical differences between chondrules, physical differences may have played a role in determining their textures. Figures 6 and 7 demonstrate the effects that radius has on the thermal evolution of chondrules in a given shock wave. Figure 6 shows the thermal evolution of the chondrules described in Figs. 1 and 2 as a function of distance after entering the same shock wave discussed earlier in this work. The smaller chondrule reaches its peak temperature before the larger chondrules. It then begins to cool as it slows down with respect to the gas, while at the same time the larger chondrule is still being heated as it continues to move through the gas. As the largest chondrule begins to slow and cool, its temperature is kept above that of the smaller chondrules for ~400 km before it begins to approach the temperature of the background radiation field. Because each chondrule would have to exist at the same distance behind the shock front in order to collide, compound chondrules formed in this zone would contain components that were at different temperatures (and degree of melt), possibly leading to one chondrule initiating nucleation in the other (Connolly et al. 1994).

The differences in the thermal processing of the chondrules can also be seen in Fig. 7, which shows the cooling rates of the chondrules in the shock. While the

chondrules reach nearly the same peak temperatures, the smaller chondrule is heated and cools more rapidly than the larger ones because of its lower mass and the fact that it loses its relative velocity (and heat input from gas drag) over a shorter period of time. Thus the smaller chondrule cools ~2 times faster than the largest chondrule after reaching its peak temperature. While a difference of a factor of two in the cooling rate of the chondrules is not necessarily large, it may still have played a role in allowing chondrules in the same shock wave to develop different textures. This would especially be true if the difference in size of the chondrules were combined with differences in heat capacity or density as might be expected if the chondrule compositions varied. Thus there are many reasons why two chondrules that were processed by the same shock wave may have formed different textures, ranging from the chemical compositions of the precursors to their sizes. If the more rapid cooling does aid in the formation of non-porphyritic textures, then the calculations shown here could explain why the secondary chondrules in compounds observed by Wasson et al. (1995) were more likely to be smaller than their primaries and have non-porphyritic textures.

#### DISCUSSION

In studying compound chondrules, Wasson et al. (1995) found that the smaller of the components in a compound chondrule was on average ~0.3 times the size of the larger. Such a significant size difference between the components is consistent with collisions being more common between different-sized particles than like-sized particles. In the case of relative velocity due to turbulence, the collisional velocities can also depend on the size difference between the particles (Cuzzi and Hogan 2003). However, even in the case

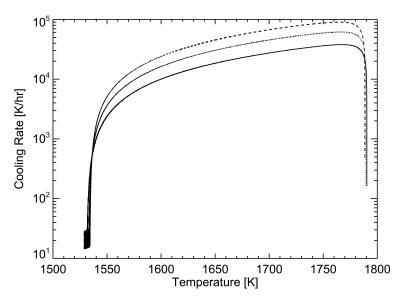


Fig. 7. The cooling rates of the chondrules described in Fig. 7. The simulation was halted when the chondrules reached temperatures of  $\sim$ 1530 K, at which point they would cool at the same rate of  $\sim$ 10 K/h.

of a very turbulent disk, for example with a turbulence parameter of  $\alpha=10^{-2}$ , the relative velocity between all chondrule sized particles will be on the order of 10 cm/s. The shock wave model for chondrule formation provides a natural way by which a wide range of relative velocities between chondrules can develop and be a strong function of the differences in size. Thus, the observations of compounds in primitive meteorites and the predictions for collisional evolution of chondrules behind a shock wave are consistent with one another.

In addition, the shock wave model predicts that compound chondrules whose components exhibit a large amount of deformation should be less numerous than those that exhibit minor deformation. As shown in Figures 3 and 4, chondrules will deform more when they have low viscosities and collide at high velocities. These conditions will be most easily met immediately behind the shock front when the chondrules are closest to their peak temperatures (less viscous state) and have the highest relative velocities between them. As the chondrules move downstream, they will initially cool very rapidly (>10<sup>4</sup> K/h) (Desch and Connolly 2002; Ciesla and Hood 2002) and become more viscous. They will then cool more slowly at lower temperatures, spending a longer period of time in this more viscous state. The number of collisions a typical chondrule would experience is proportional to the product of the relative velocity between the chondrules, their number density, and the time that relative velocity exists ( $N \propto vnt$ ). Immediately behind the shock wave, when chondrules have their lowest viscosities, the relative velocity between the chondrules will be  $10^3$ –  $10^4$  cm/s for a period of ~100 s, and the number density will be roughly the same as it was in the pre-shock gas. (We exclude velocities of  $\sim 10^5$  cm/s because those collisions are likely to be disruptive as shown above.) After the particles

experience their rapid cooling, their relative velocities will have decreased to 10-1000 cm/s and would be susceptible to collisions for a period of 10<sup>3</sup>–10<sup>4</sup> s. This time estimate comes from the period during which chondrules are expected to be above their solidus temperatures as calculated in shock wave models (Ciesla and Hood 2002; Desch and Connolly 2002). These relative velocities are due to the different stopping times of the particles as well as the random velocities that are expected to arise in a turbulent protoplanetary disk (Cuzzi et al. 2001). Over this time period, the spatial concentration of chondrules is expected to have increased by about a factor of 10 over what it was in the pre-shocked gas because the chondrules will again be moving with the gas but at with a velocity that is approximately 10 times less than that with which they entered the shock. Thus the collisional rate of the chondrules would be 1-100 times greater during the extended cooling phase in a shock wave than during the initial cooling period behind the shock wave. The collisions at this later time period will, on average, be less energetic and the chondrules more viscous, leading to less deformation of the chondrules. This is consistent with the findings by Ciesla et al. (2004a), who argued that compounds with small contact angles between the components (small deformation) are observed to be much more common than those with large contact angles.

While the observed geometries of compound chondrules appear to be consistent with the collisional evolution expected behind shock waves, estimates of collision frequency needs to be re-examined. As discussed above, only a fraction of chondrule collisions are expected to lead to the formation of compounds, with some collisions leading to bouncing or shattering. Thus estimates of chondrule densities in chondrule formation events can only be taken as absolute minimums, and that chondrule formation likely took place in regions with higher concentrations of precursors than previously thought.

Recently, Cuzzi and Alexander (2006) suggested that precursor number densities of at least 10/m<sup>3</sup> were needed to limit isotopic fractionation during the chondrule formation event. This is roughly twice the estimate predicted by Ciesla et al. (2004a), who assumed perfect sticking in chondrule collisions. Actual values of the sticking efficiency are difficult to estimate because, as has been discussed here and demonstrated in experimental work, collisional outcomes are dependent on the Weber number of the particles, the viscosity of the liquid, the relative sizes of the particles, the impact parameter, and the density of the ambient medium (Ennis et al. 1991; Rein 1993; Liu et al. 2000; Willis and Orme 2003). Experiments are needed to determine the specific criteria needed for compound chondrules to form. If the sticking efficiency is on the order of 50% or less, then the compound chondrule record and isotopic record in chondrules are leading to similar constraints, and both favor large-scale shock waves as the process responsible for chondrule formation.

While the focus of this work has thus far been on whether chondrules would survive collisions in shock waves, disruptions are expected to have occurred as well. Chondrule fragments are one of the main components of the matrix in chondritic meteorites (Brearley 1996; Alexander 2005) demonstrating that not all chondrules survived the collisions they experienced, whether they were during a chondrule formation event or not. While some of these fragments may result from collisions between the meteorite parent body and some impactor, some chondrule fragments have fine-grained rims similar to whole chondrules which may have been accreted as the core object moved through the solar nebula gas (Metzler et al. 1992; Cuzzi 2004). It is possible that such fragments were the result of collisions within the nebula itself, rather than on a parent body (Ueda et al. 2001). Further work is needed, but some amount of energetic collisions may be necessary in order to explain many of the other features observed in chondritic meteorites.

Finally, while it has been shown that the high-velocity collisions of chondrules do not pose a problem for the shock wave model for chondrule formation, and in fact that many of the predictions match the published meteorite observations, more work is needed. As has been shown here, the viscosity of the chondrules during their molten state will play a critical role in determining the outcome of collisions, so experiments should be done to examine how chondrule melt viscosity changes as a function of temperature, composition, and degree of melt. In addition, most of the observations of compound chondrules in meteorites have been done in thin sections, which look at random slices of a meteorite. As discussed by Ciesla et al. (2004a), these studies reduce three-dimensional objects to two dimensions, leading to possibly significant information being lost (such as compound chondrule abundances being underestimated by a factor of  $\sim 5$ ). Three-dimensional studies of the physical structure of compound chondrules and their abundances in

meteorites are needed in order to fully decipher the meteoritic record. X-ray tomography may be applied to provide the needed information (Kuebler et al. 1999; Hylton et al. 2005). Such analyses may provide better statistics on the number of compound chondrules in a given meteorite as well as the deformation that each chondrule experienced during the collision that formed them. Lastly, experimental work is needed to test the predictions of the modeling done here. Thus far, experimental reproduction of compound chondrules has focused on low velocity collisions (Connolly et al. 1994), but high velocity collisions should also be investigated.

#### **SUMMARY**

If shock waves were responsible for the thermal processing of chondrules, large relative velocities (~1 km/s) could develop due to differences in the sizes of the particles. It has been suggested that these velocities would result in the destruction of chondrules (Kring 1991; Nakamoto and Miura 2004; Uesugi et al. 2005). This is likely true in the cases where the chondrules are totally molten and near their peak temperatures so that the viscosity of the melt is low. Here it has been shown that as chondrules cool they can survive collisions with one another at velocities up to 100 m/s due to viscous dissipation in the melt. In chondrule-forming shocks that occurred in the central scale height of the nebula (Desch and Connolly 2002; Ciesla and Hood 2002), the amount of time that relative velocities would exist above this threshold would be approximately 100 s, during which only a small number of collisions would occur. After this time period, collisions would still occur, but the viscous chondrule melt would be able to dissipate the energy in the collision, allowing the chondrules to survive. Even when high-velocity collisions take place between chondrules, the end result is not necessarily total destruction, but more likely a number of chondrules of different sizes. In addition to having different dynamic histories, chondrules of different sizes would have been heated and cooled at different rates in a given shock wave. This could lead to different chondrules developing different textures despite being processed by the same shock wave. This allows compound chondrules whose components have different textures to form from objects that were processed in the same shock wave. Thus, the expected collisional evolution of chondrules in shock waves for chondrule is consistent with the observed properties of compound chondrules.

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