Meteoritics & Planetary Science 39, Nr 11, 1859–1868 (2004)
Abstract available online at http://meteoritics.org

A note on the snow line in protostellar accretion disks

M. PODOLAK* and S. ZUCKER†

Department of Geophysics and Planetary Sciences, Beverly and Raymond Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel
†Present address: Observatoire de Geneve, 51 Ch. des Maillettes, 1290 Sauverny, Switzerland
*Corresponding author. E-mail: morris@post.tau.ac.il

(Received 21 March 2004; revision accepted 5 July 2004)

Abstract–The temperature of ice grains in a protostellar disk is computed for a series of disk models. The region of stability against sublimation is calculated for small ice grains composed of either pure ice or "dirty" ice. We show that in the optically thin photosphere of the disk the gas temperature must be around 145 K for ice grains to be stable. This is much lower than the temperature of 170 K that is usually assumed.

INTRODUCTION

Several years ago, Podolak and Mekler (1997) presented computations of energy balance for ice grains in a Hayashi-type solar nebula. They calculated the energy fluxes due to absorption and emission of radiation, contact with the ambient gas, and condensation and evaporation of water vapor. They showed that if the grains were composed of pure water ice, then there were regions of the nebula sufficiently far from the Sun where the grains would be stable against sublimation. This was due to the fact that water ice is a very weak absorber of visible radiation, and at large enough distances solar heating is not effective. If there is an admixture of material that absorbs in the visible ("dirt"), however, small grains always absorb sufficient sunlight so that the rate of sublimation exceeds the rate of recondensation of water vapor from the surrounding nebula. For sufficiently large distances and sufficiently large (or small) grains, the sublimation rate is very small, and the grains can easily survive for the lifetime of the solar system. But they cannot grow by condensation. This conclusion was troubling, since it did not allow grains to ever grow unless they were composed of pure ice. To examine how general this result is, we have recomputed the energy balance of such grains for a more realistic nebular model.

The nebular model is that of Bell et al. (1997). It is based on the assumption of a constant mass flux through the disk and an α-law for viscosity. The models computed by Bell et al. approximate the time evolution of the disk by calculating quasi-steady state models for different values of the mass flux. These values range from 10^{-5} $M_\odot$ yr^{-1}, corresponding to a very young and active disk, to 10^{-9} $M_\odot$ yr^{-1} corresponding to a relatively quiescent late stage of evolution. These models are very similar in structure to those computed by other groups (Sasselov and Lecar 2000; Richard, personal communication). Our calculations were done for a nebular model with $\alpha = 10^{-4}$ and $\dot{M} = 10^{-8}$ $M_\odot$ yr^{-1}. This is typical of the parameters expected for a disk at the later stages of planet formation. The conditions during the beginning of planet formation and of meteor formation are probably better represented by models with a higher mass flux. Further details can be found in the reference cited.

Given the local temperature and gas density, as well as the local radiation field, it is possible to compute the energy balance for different heating and cooling processes in an ice grain. The details are given in Mekler and Podolak (1994) and Podolak and Mekler (1997), but we present the basic equations here for the convenience of the reader. There are two major processes that heat the grain. One is absorption of radiation from the ambient field. If $S(\lambda)$ is the rate of energy production by the central star, then if the optical depth to the star is low (i.e., we are in the photosphere of the nebular disk), then the radiative energy absorbed by the grain per unit area per unit time is given by:

$$E_{rh} = \frac{1}{4R^2}\int_0^\infty Q_{abs} S(\lambda) d\lambda \tag{1}$$

where R is the distance from the star, and Q_{abs} is an efficiency factor for absorption of radiation that multiplies the geometric cross section of the grain (see below). If we are in an optically thick region of the disk, then Equation 1 must be replaced by:

$$E_{rh} = \int_0^\infty Q_{abs} B(\lambda) d\lambda \tag{2}$$

where $B(\lambda)$ represents the local wavelength-dependent radiation field. In our computations, we approximated this by taking the black body radiation at the ambient temperature. The efficiency factor for radiation absorption is computed for the grain based on its (complex) index of refraction and its size relative to the wavelength of the radiation (Mie extinction).

The refractive index for water is taken from the work of Warren (1984), and the refractive index for "dirt" is assumed to be that appropriate for some generic absorber. We have used a real index of 1.45 and an imaginary index of 0.1 as representative, but the results are not very sensitive to the refractive index or to the amount of dirt mixed in with the ice. This happens because the main difference between pure water ice and a generic absorber is that the water ice is transparent in the visible light. Since the Sun radiates strongly in the visible, this has important consequences for the heating rate of an ice grain. Once some dirt is mixed into the ice that absorbs in the visible, the reaction of the grain to solar heating is strongly affected. However, adding more dirt or adding dirt with somewhat different absorption properties has a much smaller effect (Podolak and Mekler 1997).

The second heating mechanism is the impact of water molecules. These are assumed to stick to the grain and release energy in the form of latent heat. If n_{H_2O} is the number density of water molecules in the gas, then the heating rate per unit area due to water condensation is:

$$E_{cond} = \frac{n_{H_2O}}{2} q \sqrt{\frac{2kT_{gas}}{\pi m_{H_2O}}} \tag{3}$$

where T_{gas} is the local gas temperature, k is Boltzmann's constant, m_{H_2O} is the mass of a water molecule, and q is the latent heat released on condensation.

The grain cools by reradiation. The rate is given by:

$$E_{rc} = \int_0^\infty Q_{emis} F(\lambda, T_{grain}) d\lambda \tag{4}$$

where $F(\lambda, T_{grain})$ is the Planck function for the particular grain temperature T_{grain} and Q_{emis} is the efficiency factor for emission. It can be shown that $Q_{emis} = Q_{abs}$ (Bohren and Human 1983). The second cooling mechanism is due to sublimation of water molecules from the grain surface. The cooling rate for this process is given by:

$$E_{sub} = q \frac{P_{vap}(T_{grain})}{\sqrt{\pi m_{H_2O} k T_{grain}}} \tag{5}$$

Here, P_{vap} is the vapor pressure of the ice. In our model, we used the vapor pressure formula given in *International critical tables of numerical data, physics, chemistry and technology* (1937). This formula was originally meant to fit the measurements of vapor pressure over crystalline ice at temperatures down to 180 K, but more recent measurements show that it can be safely extended down to 145 K (Notesco 1990). This is the region that is important for the calculations presented here. Since ice that is deposited in this temperature range will be crystalline and not amorphous, we have not considered the latter form of ice in this work.

The third mechanism that can either heat or cool the grain is contact with the gas. The energy transfer rate to the grain is given by:

$$E_{gas} = \frac{n_{H_2}}{4} \sqrt{\frac{8kT_{gas}}{\pi m_{H_2}}} \frac{jk(T_{gas} - T_{grain})}{2} \tag{6}$$

where we have assumed that the bulk of the gas is H_2, and j is the number of molecular degrees of freedom ($j = 5$ for H_2). If the gas is hotter than the grain, energy is transferred to it and the grain is heated. Otherwise, E_{gas} is negative, and the grain is cooled.

The sum of all these terms gives the net heat transfer to the grain. In steady state, this must be zero so that T_{grain} can be found by iteration. Since the rates of heating and cooling depend on the size and composition of the grain, different grains in the same region can have different steady state temperatures. As a result, it is not sufficient to simply equate the grain and gas temperatures and to assume that some particular gas temperature always defines the snow line.

MIDPLANE

The midplane of the disk is characterized by gas that is optically thick to the radiation from the star. The radiation reaching the grain is, therefore, radiation at the wavelength determined by the local gas temperature. Figure 1 shows typical grain behavior and can serve as an introduction to the type of effects one can expect. The energy fluxes (per unit area of grain surface) of the various heating (solid curves) and cooling (dotted curves) terms are shown as a function of grain distance from the star. In Figs. 1, 2, 4, 5, and 6, the term "radiative heating" refers to the heating rate described by either Equation 1 or 2. The term "H_2O heating" refers to the heating rate described by Equation 3. Equation 4 is given by the term labeled "radiative cooling," Equation 5 by "H_2O cooling," and Equation 6 by "gas heating" when it is positive and "gas cooling" when it is negative.

The stellar surface is assumed to have the temperature of 6000 K, and the calculation is done for 10 μm pure ice grains. Far from the star (>3 AU), the grains are essentially at the same temperature as the gas because the grain temperature is determined mainly by physical contact with the gas. In Fig. 1, it would appear that radiative processes control the temperature since the contribution due to gas heating is orders of magnitude smaller than radiative absorption from the ambient gas and reradiation from the grain. But this is misleading. The heat flux into the grain from the contact with the gas is low because the grain is at nearly the same temperature as the gas. As will be shown below, the gas fixes the grain temperature. Inward of

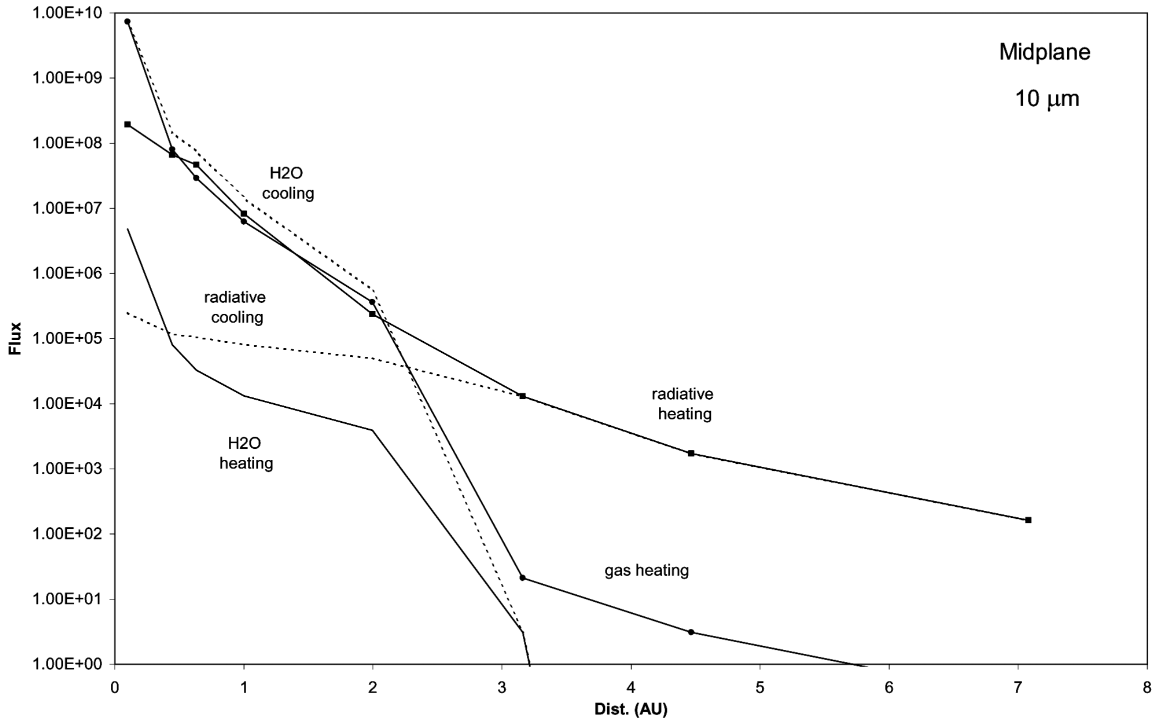

Fig. 1. Energy flux due to heating (solid) and cooling (dashed) by various sources for a 10 μm pure ice grain. The curve for gas heating is marked by diamonds to distinguish it from the radiative heating curve (squares) which nearly overlaps for radii less than ~3 AU. Further details of the model are given in the text.

3 AU, the gas temperature increases sharply. At temperatures above ~150 K evaporative cooling becomes important and quickly becomes the dominant cooling mechanism. Due to its efficiency, evaporative cooling easily counters the rise in radiative heating and (because the grain now stays noticeably cooler than the surrounding gas) in gas heating, and ensures that the grain temperature never rises above T ~300 K. At first sight, it seems strange to have ice grains at the temperature of T <300 K in contact with gas of T >1500 K, but this means that the grains survive for a very short time—a fraction of a second.

Figure 2 shows the same energy balance calculation for a 0.1 μm grain. The cross section for the interaction between a grain of radius *a* and radiation is given by:

$$\sigma = Q\pi a^2 \tag{7}$$

where Q is an efficiency factor that depends on the properties of the grain and the wavelength of the radiation. For spherical grains, where Mie theory applies, Q depends on the complex index of refraction of the grain material and on the ratio of the grain radius to the wavelength of the radiation. This ratio is characterized by the size parameter of the grain which is defined as:

$$x = \frac{2\pi a}{\lambda} \tag{8}$$

Typically, Q is close to 1 for x ≥1 and decreases rapidly for smaller values. At 3 AU, where both the grain and the gas temperatures are near 150 K, the wavelength of both the absorbed and emitted radiation is λ ~20 μm. A grain with a = 10 μm will have x ≅ 3 and the grain will absorb and emit radiation efficiently. However, a grain with a = 0.1 μm will have x ≅ 0.03, and will have a very small cross section for both emission and absorption of the radiation characteristic of T = 150 K. This can be seen in Fig. 1 where, for distances greater than 3 AU, the radiative heating cooling fluxes for the 0.1 μm grain are one to two orders of magnitude smaller than for the 10 μm grain. Even so, the temperatures of the two grains are nearly identical, so contact with the gas must be the controlling factor.

Since water ice absorbs well in this wavelength region, the absorption and emission properties of the grains are insensitive to the details of the index of refraction and, thus, to the presence of dirt. Figure 3 shows the temperature of the gas at midplane as well as the temperatures of four different grains: 10 μm radius, pure ice; 10 μm radius, dirty ice; 0.1 μm radius, pure ice; and 0.1 μm radius, dirty ice. The grain temperatures are so close to each other that the curves are nearly indistinguishable. Close examination shows that the dirty ice grains of both 10 μm and 0.1 μm (solid curves) are somewhat hotter than the corresponding pure ice grains (dashed curve). However, the different size grains have nearly identical tempertures. The calculations were carried out for the case where the absorber was present with a mass fraction of 0.1, but as noted, the results are insensitive to the precise value. For all these cases, the snow line where the ice grains first become stable falls where the temperature of both the grains and the gas is 143 K. For the

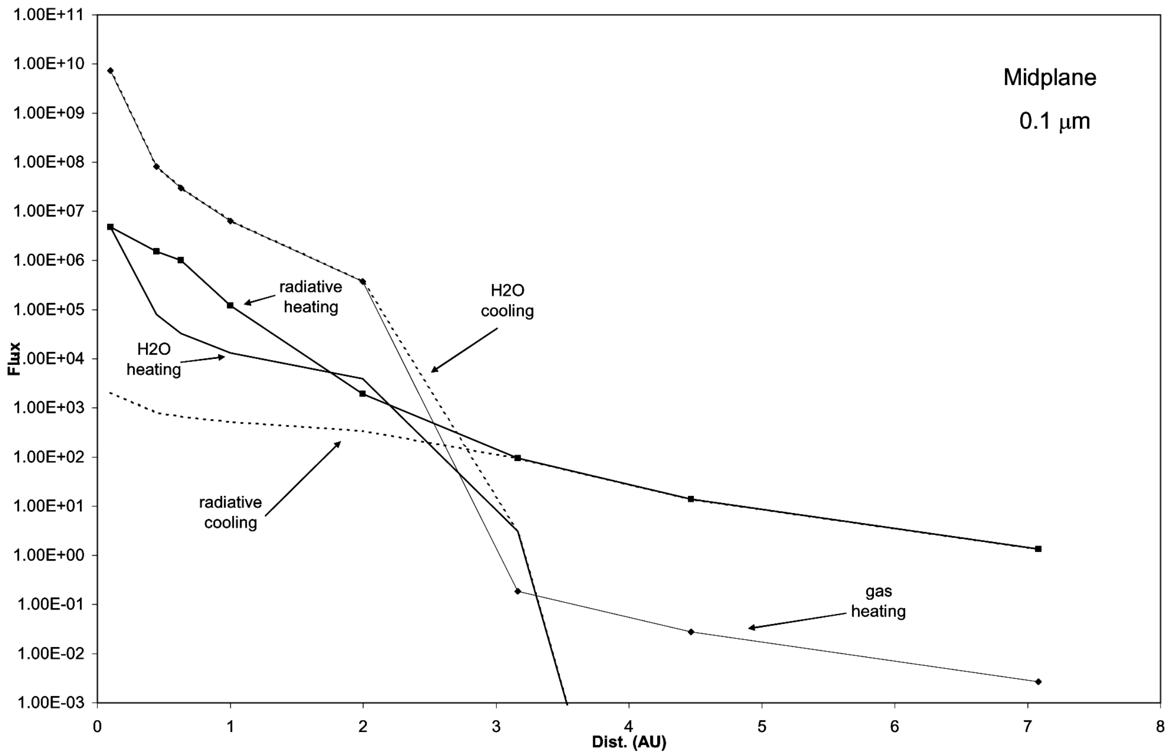

Fig. 2. Same as Fig. 1, but for a 0.1 μm pure ice grain.

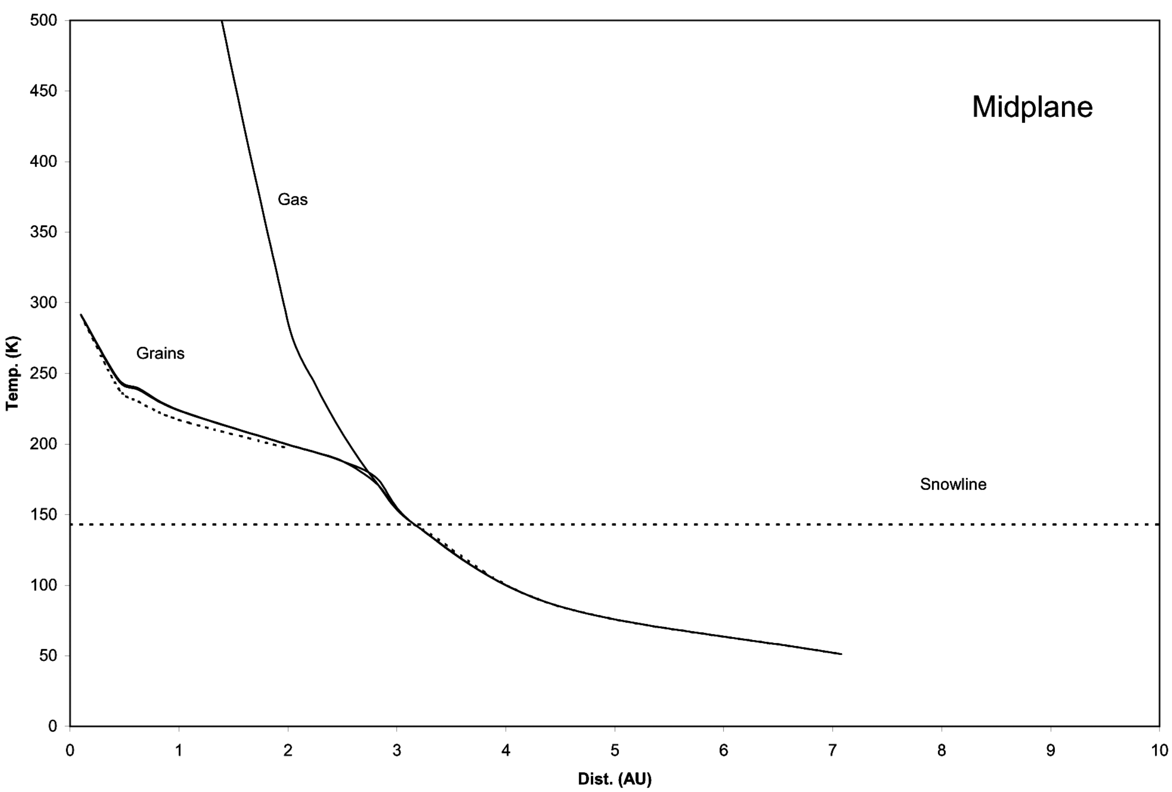

Fig. 3. Temperatures of gas and grains at midplane. 10 μm and 0.1 μm grains of both pure (dashed curve) and dirty (solid curve) ice are shown, but the curves are nearly indistinguishable. The dotted horizontal line indicates the temperature of 143 K. This is where the grains begin to be stable against evaporation.

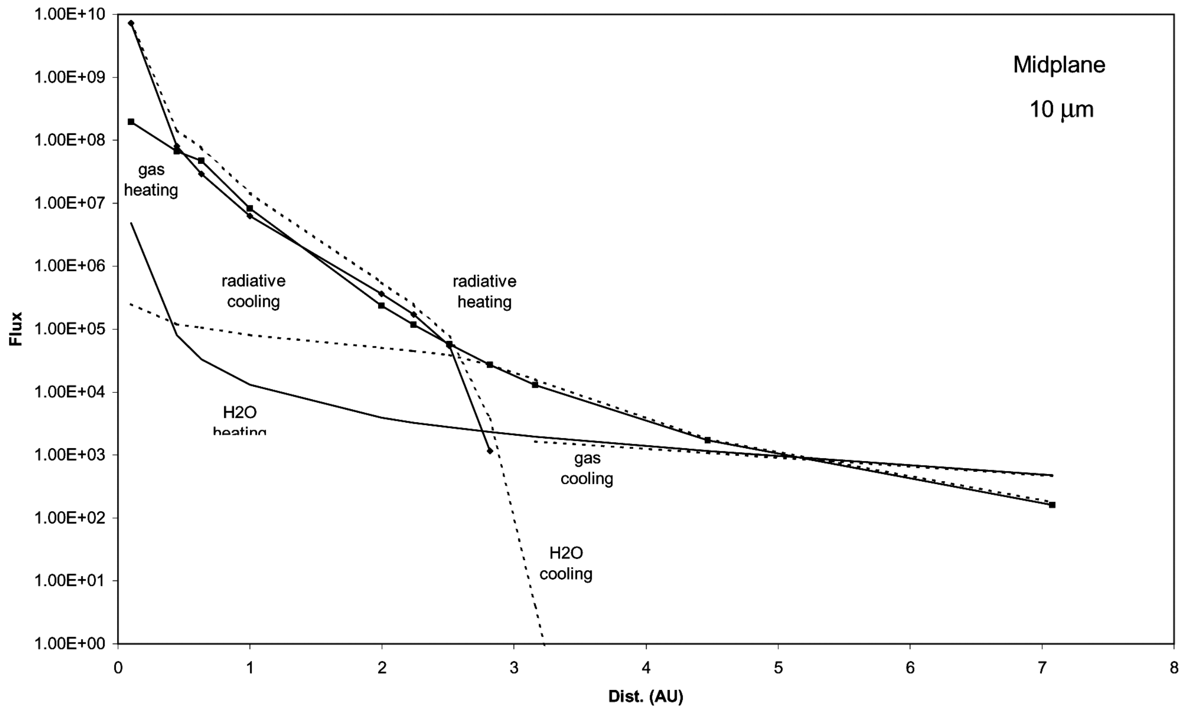

Fig. 4. Same as Fig. 1, but with the background water vapor always equal to solar value. Note that beyond ~3 AU gas heating ceases and is replaced by gas cooling (dotted curve). This cooling is almost exactly equal to the heating by water vapor condensation (solid curve). The slight gap between the gas heating and gas cooling curves is an artifact of the finite resolution of the nebular model.

particular nebular model investigated, this occurs at a distance of nearly 3.2 AU from the central star. This is in contrast to the usual assumption (see e.g., Sasselov and Lecar 2000) where the snow line is put at 170 K. In the nebular model studied, this latter point occurs at ~2.8 AU.

The computations described till now were performed with the assumption that the partial pressure of the water vapor in the nebula is always equal to the saturation vapor pressure of ice at that temperature, as long as the number fraction of water vapor molecules remained below 7×10^{-4} which is roughly the solar H_2O/H_2 ratio. The water vapor mass fraction was never allowed to exceed this value. Increasing this maximum value to 1×10^{-3} has basically no effect on the results. If, however, we keep the $H_2O/H_2 = 7 \times 10^{-4}$ in the gas independent of temperature (such as might happen if a plume of warm, moist, gas was suddenly mixed in with colder gas), there is some small effect. Figure 4 shows the heat balance for 10 μm grains for this case. The major difference is that beyond 2 AU the heating of the grain by H_2O condensation is orders of magnitude higher because of the much higher water vapor abundance. For the most part, this is offset by the radiative cooling, but the grain temperature rises to just a little above the ambient gas temperature and the contact with the gas removes the remaining excess heat. Again, we see that the contact with the gas is the main factor that determines the grain temperature. The actual difference in the temperature of the grain for the two cases is less than a degree. The point where the H_2O heating and the H_2O cooling are equal is the position of the snow line, and, because of the increased H_2O condensation, it now falls at ~2.8 AU where both the gas and the grains have a temperature of ~170 K. This is more in accordance with traditional estimates of the position of the snow line.

PHOTOSPHERE

The outermost, optically thin, layer of the disk (photosphere) is directly exposed to stellar radiation and here the energy balance is quite different. The gas density is much lower and radiative processes are more important. The photosphere has a low optical depth for stellar radiation. As a result, the grain temperature (and the position of the snow line) will depend critically on the grain size and composition. Starlight will be the major source of radiative heating and this will be in the visible wavelengths. Both 10 μm grains and 0.1 μm grains will have $x \geq 1$ and will absorb radiation efficiently. But at the relevant grain temperatures (≤300 K), the 0.1 μm grains will have $x \ll 1$ for emission and will radiate inefficiently. Therefore, we would expect small grains to be at a higher temperature than large grains. Since pure water ice is transparent in the visible, pure water ice grains will not absorb efficiently in this wavelength regime. Thus, in addition, we would expect different temperatures for pure and dirty ice grains, respectively.

The various heating and cooling terms for 10 μm pure ice grains are shown in Fig. 5. Radiative heating is the main source of energy input and is balanced by evaporative cooling inside of ~0.5 AU, by radiative cooling from 0.5 to ~3.2 AU, and by contact with the gas beyond this point. The grain temperature is always higher than the gas temperature and, as a result, the rate of evaporation from the grain is always higher than the rate of recondensation. These grains are not stable in the photosphere at any distance from the star, and, in this sense, there is no snow line in the photosphere.

Far enough from the star, where the grain temperature is sufficiently low, the grain will still be evaporating, but the rate will be so low that its lifetime, defined by

$$\tau_{grain} = a\frac{dt}{da} \tag{9}$$

will be so long that a significant change in the grain radius will not occur within the age of the solar system. This leads to a possible alternative definition of the snow line in the photosphere: the distance at which the grain lifetime exceeds the lifetime of the gas disk. A conservative estimate of this latter value would be 10^7 yr = 3×10^{14} sec. If this is taken to define the snow line, then for this particular case, the snow line occurs at ~1.8 AU. Interestingly, the gas temperature in this region is T_{gas} ~50 K. This is much lower than what is canonically taken to define the snow line.

Figure 6 shows the same calculation for 0.1 μm pure ice grains. Here, radiative heating is the major energy input term, and outward of 1 AU it is balanced by cooling as a result of contact with the gas. It is only inward of 1 AU that the gas can be hotter than the grains, and at this point the contact with the gas heats the grains. This is not directly shown in the figure, but can be seen in the downturn of the gas cooling curve, and in the fact that inward of ~0.5 AU, the evaporative cooling term is larger than the radiative heating, though there appears to be no additional heat source. In fact, the additional heat source is the gas heating, but this occurs in such a small range of radial distance that it is not shown. Note also that the radiative heating per unit area on a smaller grain is more than an order of magnitude lower than the heating of a larger grain. This is due to the fact that the larger grain has a much greater absorption efficiency in the visible as a result of its larger size parameter. The 0.1 μm grains are warmer than the surrounding gas at all distances greater than ~0.5 AU, and so are unstable against evaporation at all heliocentric distances. Again, the snow line may be defined as the point where the lifetime of the grain is longer than the lifetime of the gas disk. Since the lifetime, as defined by Equation 9, is proportional to the grain radius, smaller grains have larger lifetimes because their reduced efficiency of absorption gives them lower temperatures. The snow line for 0.1 μm grains is, thus, closer to the star, at ~1 AU, where gas temperature is ~80 K.

The presence of impurities that absorb in the visible will also affect the absorption properties of the grains in the photosphere. In general, the dirt makes both the large and small grains better absorbers (and emitters), their temperatures increase, and the snow line is moved further from the star. Figure 7 compares the lifetimes of pure and dirty ice grains of 0.1 μm (dashed curves) and 10 μm (solid curves) radii in the photosphere. The "dirt" comprises a mass fraction of 0.1. The grain temperatures for these cases are shown in Fig. 8. It can be seen by comparing Figs. 7 and 8, that the photospheric snow line always falls at a grain temperature of ~130 K. This is quite close to the grain temperature found for the midplane snow line.

Since refractory particles can act as nuclei for ice condensation, it is interesting to ask what the difference is between the dirty ice grains that we have been discussing and the case where the "dirt" is concentrated at the center of an ice grain. Figure 9 shows the temperature of grains in the photosphere as a function of radial distance for 10 μm grains composed of pure ice, dirty ice with the absorber comprising 10% of the grain mass, a pure ice grain with a 1 μm core, and a pure ice grain with a 5 μm core. For an absorber with a density of 2.5, which is typical of a silicate-organic mix, the 1 μm core represents an absorber mass fraction of 2.5×10^{-3}, while the 5 μm core represents an absorber mass fraction of 0.26. As can be seen in the figure, the grain with the 5 μm core is cooler than the grain where the absorber is only 0.1 of the total mass, but is spread over the entire volume. The overall behavior is similar, however. The square on each curve marks the location of the snow line for this particular case. Again, the grain temperatures are in the 150 K range. Also shown (dotted curves) are the grain temperatures for 10 μm grains at midplane. One curve is for the pure ice case, while the other is for a grain with a 1 μm core. In all cases, the H_2O/H_2 ratio in the gas was assumed to be solar.

A curious feature can be seen in Fig. 10. We have plotted the position of the snow line in the photosphere for the case where the nebular water vapor abundance is kept at the solar value. The dashed curves show the energy flux leaving a dirty ice grain in the form of sublimated ice vapor. The solid curves are for pure ice. The heavy solid curve is the energy flux input to the grain through the condensation of water vapor from the gas. The place where the two curves cross marks the photospheric snow line for the particular grain size. The vertical dashed line shows the position of the midplane snow line for this case. Note that for both the pure ice and dirty ice grains, the 0.1 μm and 10 μm grains are stable closer to the star than the 1 μm grains. If the grains are originally small, they will not be able to grow beyond ~1 μm. If a size distribution of large and small grains occurs, as might happen if icy planetesimals underwent collisional breakup, then one could imagine a region where 0.1 μm grains will grow (as will grains of 10 μm radius), while grains of 1 μm radius will lose mass. Such a region will, therefore, develop a bimodal grain size distribution. Observations of disks generally assume a power law distribution of grain sizes to which observations are fit. However, there are some intriguing indications of bimodal size distributions in interstellar grains (Israel et al. 1999).

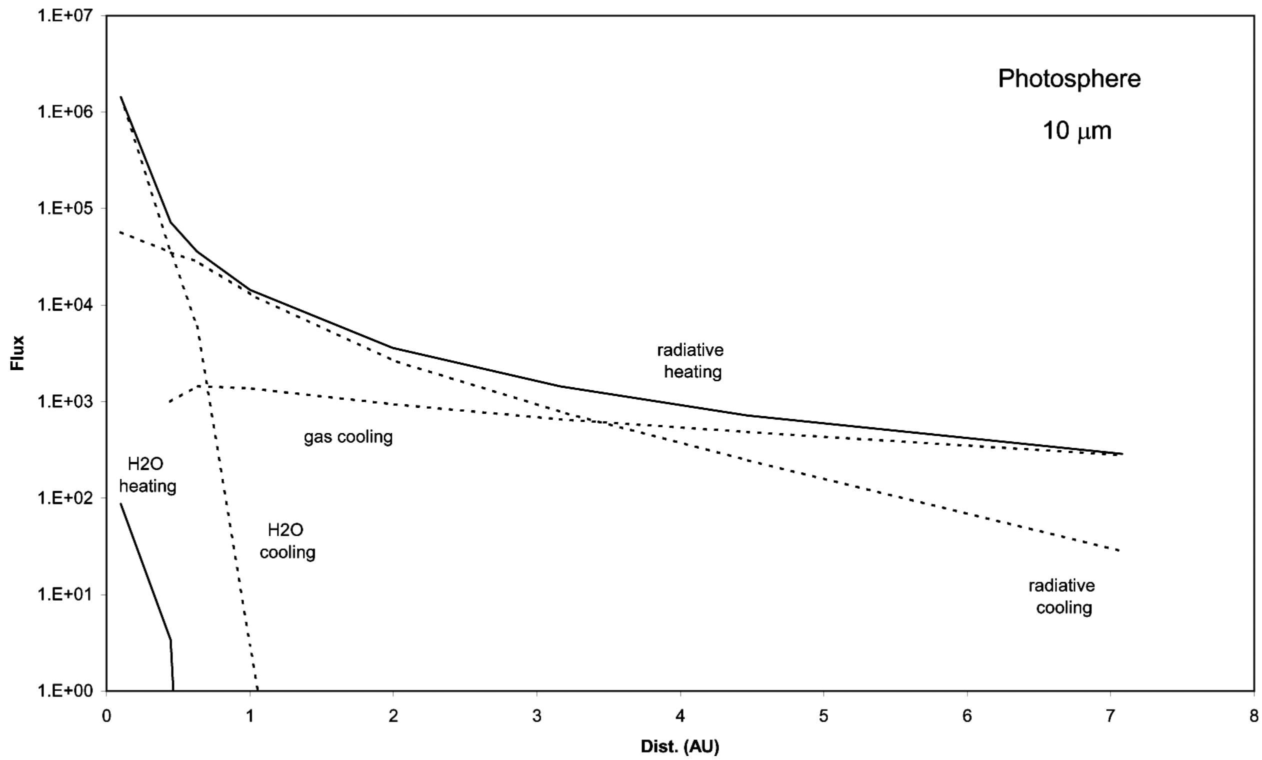

Fig. 5. Same as Fig. 1, but for 10 μm in the photosphere.

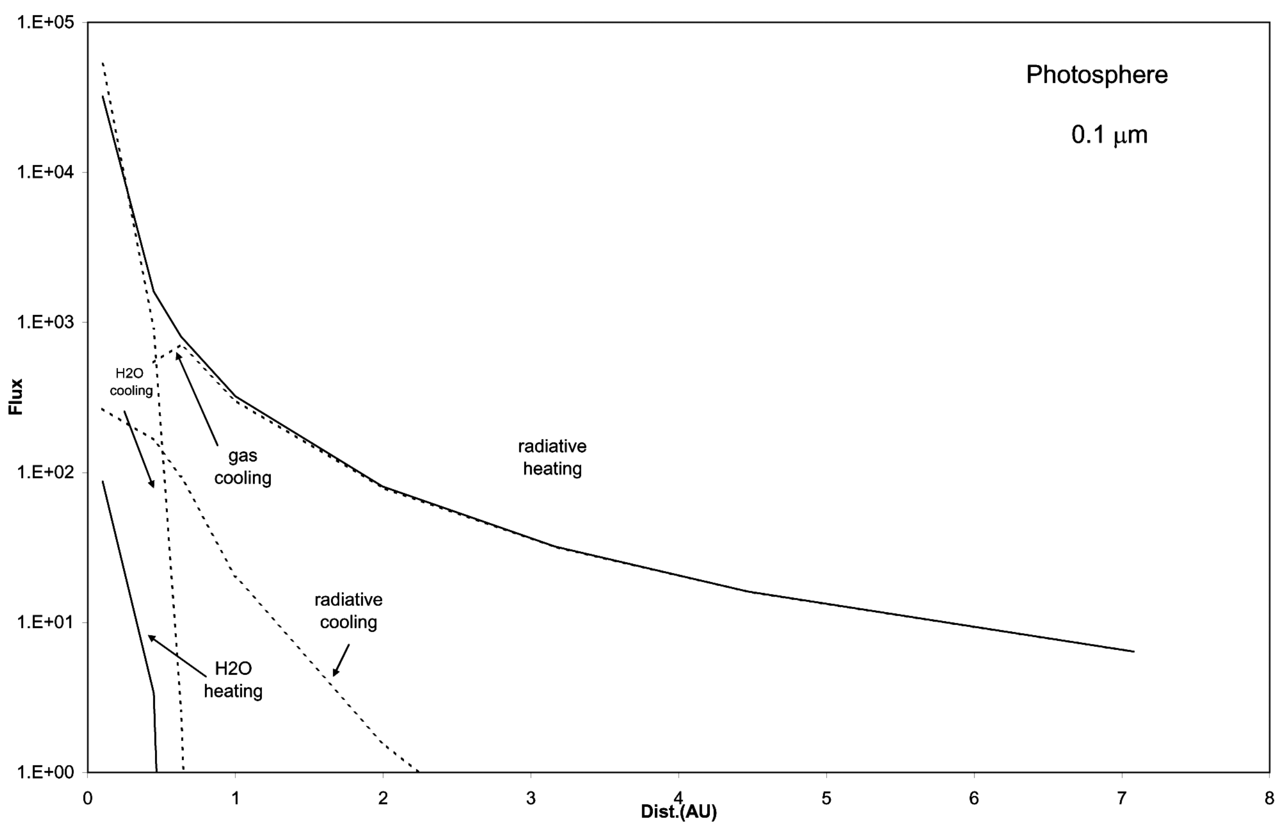

Fig. 6. Same as Fig. 1, but for 0.1 μm in the photosphere.

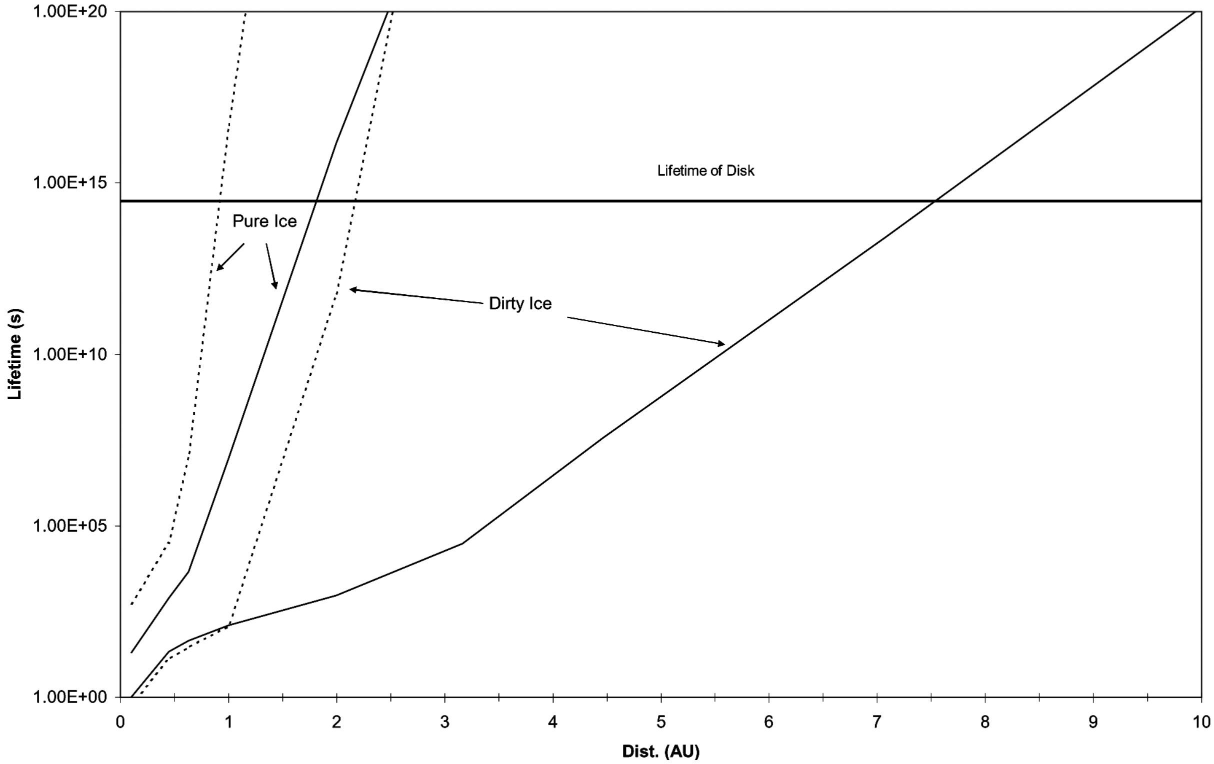

Fig. 7. Comparison of lifetimes for pure and dirty ice grains. The dashed curves are for 0.1 μm grains and the solid curves are for 10 μm grains. The solid horizontal line shows an age of 10^7 yr which is a conservative upper limit to the age of the gas disk.

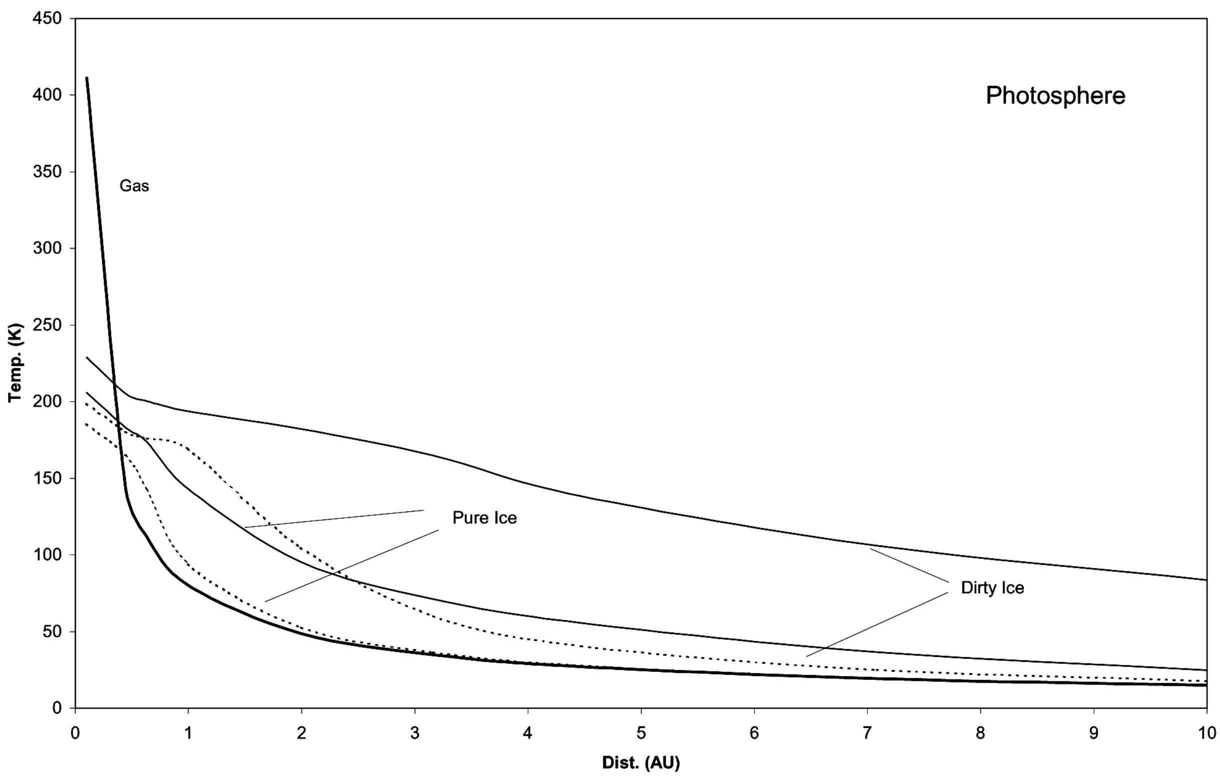

Fig. 8. Temperatures of pure ice and dirty ice grains in the photosphere. The dashed curves are for 0.1 μm grains and the solid curves are for 10 μm grains. The heavy solid curve shows the temperature of the photospheric gas.

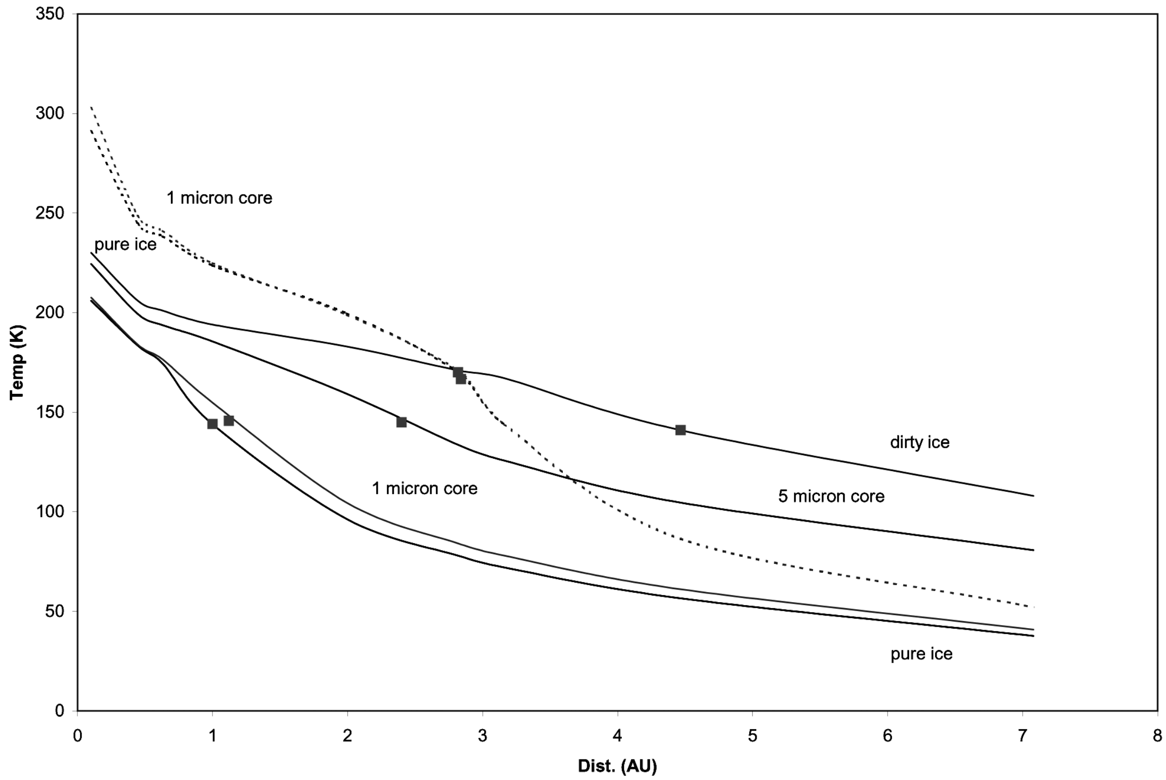

Fig. 9. Grain temperatures for 10 mm grains in the photosphere (solid curves). As marked in the gure, the grains are: pure ice, dirty ice, pure ice surrounding a 1 mm core, and pure ice surrounding a 5 mm core. Also shown are 10 mm grains at midplane (dotted curves), one of pure ice the other with a 1 mm core. The squares on the curves show the location of the snowline for each case.

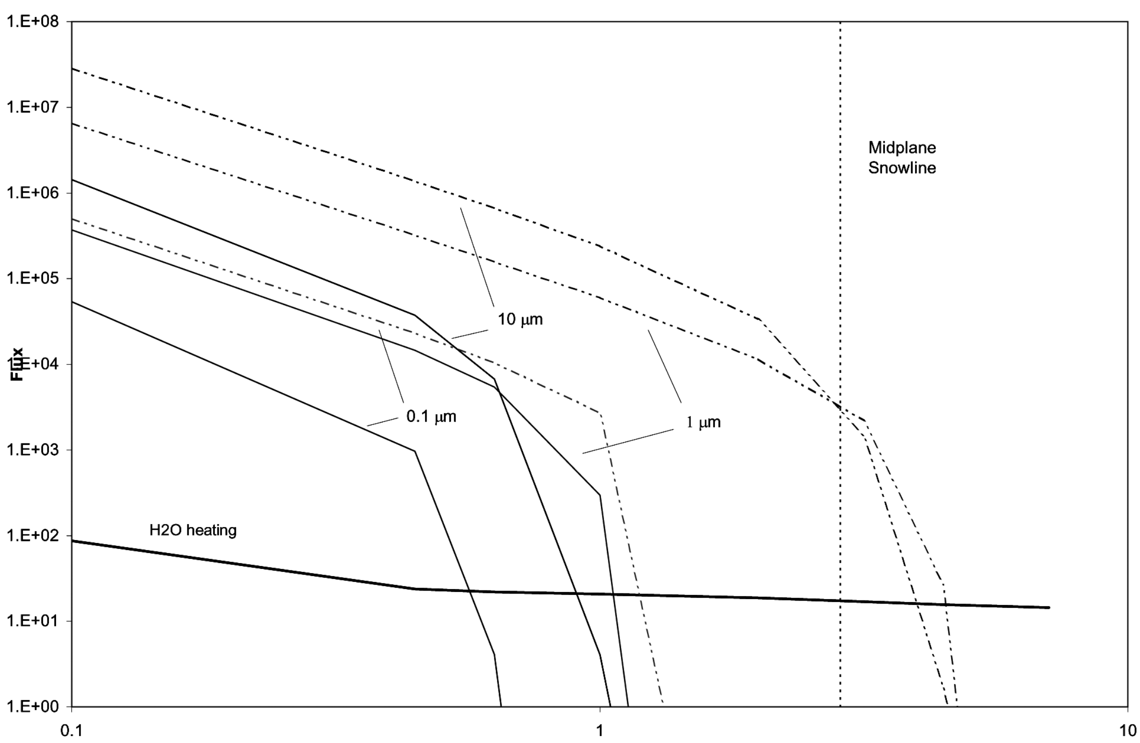

Fig. 10. H_2O cooling flux for 0.1 μm, 1 μm, and 10 μm grains in the photosphere for pure (solid) and dirty (dashed) ice. The heavy solid curve shows the heating by H_2O condensation. The snowline occurs where the heating and cooling curves cross.

DISCUSSION

The assumption that the snow line occurs at the position in the disk where the gas temperature is 170 K is often not valid. In the optically thick region of the disk (for the particular model we investigated), the snow line will develop at T_{gas} ~170 K only when the H_2O/H_2 ratio is on the order of the solar value. If the water vapor abundance is constrained by the vapor pressure at the ambient disk temperature, then the snow line will be somewhat farther from the center of the disk where the gas (and grain) temperature is on the order of 150 K. This result is nearly independent of the size and composition of the grain, provided that water ice is the major constituent and is responsible for evaporative cooling.

In addition, the classical idea of a snow line is really valid only in the optically thick region of a disk. In the optically thin region, radiation by central star will cause ice grains to evaporate at all distances in the disk. In this case, a more useful definition of the snow line is the region where lifetime of an ice grain against evaporation is greater than the lifetime of the disk. Even with this definition, the position of the snow line will be significantly different for different grain sizes and compositions, though it will always fall where the grain temperature is ~150 K.

As noted, the difference in behavior of the grains between the midplane and the photosphere comes from the difference between the stellar irradiation and the irradiation of the ambient disk gas, so it is useful to ask how sensitive our results are to assumptions about the radiation field and its interaction with the grains. We have done the Mie computations assuming that the grains are spherical. In general, this should be a good approximation even for elongated grains, provided they are randomly oriented (Wickramasinghe 1967). Other grain shapes will have surface to volume ratios larger than that of spherical grains, so for those cases where evaporative cooling is important, the steady state temperature will be slightly lower than those we found. Due to the steep dependence of vapor pressure on temperature, we do not expect these differences to be significant.

A related issue regards our assumption that the radiation is either from the star or from the gas, but not both. In reality both sources will contribute. Models of flared disks (Jang-Condell and Sasselov 2003) show that the slant optical depth in the visible to the star changes very rapidly with the height above the midplane. As a result, the region where there is significant irradiation from both the disk and the star is very thin, and one can separate the two regions to a good approximation. Our computations, in which both the visible radiation from the star and the infrared radiation from the disk were simultaneously included in the radiative heating, confirm this conclusion.

An additional consideration is the temperature of the central star. We have assumed a black body temperature of 6000 K, but other values are possible. A lower temperature would drive the black body peak toward the red, but even a temperature as low as 3000 K would still keep that peak in the wavelength region where water ice is transparent. A lower temperature will, however, greatly reduce the total flux of the radiation, and this will affect the computed temperatures for photospheric grains.

Finally, there is the question of how these results may affect disk models. The most immediate effect is that the grain opacities in such disks have to be reconsidered, at least in the photosphere where grains will not appear until gas temperatures well below 170 K are reached. In addition, since the grain opacity depends on the size of the grains, one should properly include microphysical effects such as sedimentation and coagulation to determine a realistic size distribution for these grains. Some preliminary calculations to address this issue are currently being done, and the results will be presented in a future work.

Acknowledgments–This research was funded in part by a grant from the Binational Science Foundation. S. Zucker would like to thank the Jacob and Riva Damm foundation for partial support.

Editorial Handling—Dr. Dina Prialnik

REFERENCES

Bell K. R., Cassen P. M., Klahr H. H., and Henning T. 1997. The structure and appearance of protostellar accretion disks: Limits on disk flaring. *The Astrophysical Journal* 486:372–387.

Bohren C. F. and Huffman D. R. 1983. *Absorption and scattering of light by small particles.* New York: Wiley & Sons. 530 p.

Israel F. P., van der Werf P. P., and Tilanus R. P. J. 1999. Submillimetre maps of the edge-on galaxy NGC 891. *Astronomy and Astrophysics* 344:L83–L86.

Jang-Condell H. and Sasselov D. D. 2003. Radiative transfer on perturbations in protoplanetary disks. *The Astrophysical Journal* 593:1116–1123.

Mekler Y. and Podolak M. 1994. Formation of amorphous ice in the protoplanetary nebula. *Planetary and Space Science* 42:865–870.

Notesco G. 1990. Low temperature properties of ice and applications to comets. Master's thesis, Tel Aviv University, Tel Aviv, Israel.

Podolak M. and Mekler Y. 1997. Dirty ice grains in the protoplanetary nebula. *Planetary and Space Science* 45:1401–1406.

Sasselov D. D. and Lecar M. 2000. On the snow line in dusty protoplanetary disks. *The Astrophysical Journal* 528:995–998.

Warren S. G. 1984. Optical constants of ice from the ultraviolet to the microwave. *Applied Optics* 23:1206–1225.

West C. J. and Hull C., eds. 1933. *International critical tables of numerical data, physics, chemistry, and technology.* Volumes I-VII. New York: McGraw-Hill.

Wickramasinghe N. C. 1967. *Interstellar grains.* London: Chapman and Hall. 154 p.