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MEASURING AND REPRESENTING PERIPHERAL OEDEMA AND ITS ALTERATIONS

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ABSTRACT

Correlation was very good between 1,500 simultaneous measurements of peripheral lymphoedema (arms and legs) by water displacement and by calculating volumes from circumferences, but in the legs "circumferences" gave only half the absolute amount of oedema when compared with "water displacement." For 150 arms, however, each method provided almost identical values for oedema. Arms when oedematous are fairly uniformly swollen; legs, on the other hand, are typically more oedematous distally. Circumference measurements accordingly include portions of nearly normal (i.e., minimally or nonoedematous) leg; water displacement by contrast measures only the oedematous, distal region. When only the circumferences of the lower legs were taken into consideration, the amount of oedema as measured by water displacement were almost identical. Nonetheless, measuring the proximal, more normal, or nonoedematous regions of the leg is critical for assessing treatment by physical methods (e.g., complex physical therapy).

The various equations representing oedema can be greatly affected by errors in the initial, final or normal measured volumes. Relative errors differ as these variables alter. Many of the equations are non-linear, i.e. small alterations in one variable may produce widely differing results depending on the other variables. Problems in the use of an abnormal contralateral or "normal" limb as a reference point are discussed.

The best equation to use in bilateral oedema is "Difference in Volume/Initial Volume"; in unilateral oedema the best equation is "Difference in Oedema/Normal". "Change in Oedema" i.e., "Difference in Oedema/Initial Oedema" is best derived from the Means of other equations.

There have been almost no studies of the errors and assumptions involved in measuring peripheral oedema and the results of treatment. This paper considers these issues to minimize errors inevitable in oedema measurements and to present the results in the most consistent and useful way.

The study came about because of puzzlement about why one patient had a reduction of 400% in the amount of oedema after treatment, while another with a similar contralateral normal limb and loss of volume only had a 50% reduction! It will be shown that the answer was: *the former had a normal limb of very similar volume to the initial size of the oedematous limb whereas the latter had a large difference between the two limbs.*

PART 1 – MEASURING OEDEMA

There are a number of measurements which can usefully be made of an oedematous limb — e.g. tonometry, skin thickness, skin

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temperature — but in practice the volume is the most important one. Whereas volume change may be estimated in a number of ways, the most direct is by displacement of water. To avoid errors, several considerations are worthy of note. These have been described for the arm (1, 1a) and can be similarly applied to the leg (as was done in the measurements, below). A device using beams of infrared light (Volometer, BTsl, Aachen) can estimate limb volume from the anterior and lateral silhouettes (1a).

Most simply, quickly and cheaply, the volume can be estimated from several circumference measurements (at standard distances apart) by treating each segment of the limb between each pair of circumferences as a truncated cone, including to mid-hand or foot (2). The volume of the segment is given by: $V = h \times (C^2+Cc+c^2)/(\pi \times 12)$, where: V = the volume of the segment of the limb, 'C' and 'c' are the circumferences at each end, and 'h' is the distance between them. The sum of these volumes gives a surprisingly accurate estimate (3) compared with water displacement (correlation coefficient = 0.98). However, that study (3) was only on 9 patients, so similar comparisons were made on our results.

Comparison of the results by water displacement with those by truncated cones were perfomed on legs from 1,300 simultaneous estimations in unilateral filaritic lymphoedema in India (S. Jamal, personal communication, 1986-1991). Water displacement was measured to 30 cm above the heel using the mean of two estimations, in cylinders 30 cm in diameter. Circumferences (using tapes 1 cm in width) were measured at mid-foot, at the narrowest part of the ankle, and above this site at 10 cm intervals from the heel as far up the leg as possible (usually 60 cm, since Tamil patients are short in stature); the volume was calculated from mid-foot to the top circumference, as outlined above. Both the affected and contralaternal normal limbs were measured identically. Oedema was calculated from: (volume of affected leg – normal leg)/normal leg.

The correlation coefficient between the two methods was very close (=0.934, $p<<0.0001$). However, the regression line for oedema estimated from the circumferences was:

=0.4997 × (oedema by water displacement)-0.0188 (Standard Errors, 0.0078 and 0.00023, respectively.)

Thus, the two methods were equally valid for measuring oedema, but the circumference method gave only half the amount of oedema as estimated by water displacement.

A similar study of 200 unilateral lymphoedema legs in Australia (M. Mason, personal communication, 1989-1992) also gave a correlation coefficient (0.958), but again the regression equation for oedema from the circumferences = 0.570 × (oedema by water displacement)+0.005 (S.E.'s: 0.0032 and 0.001, respectively). Here, water displacement was to 50 cm (usually 40 cm, depending on the length of the leg and the width of the thigh); circumference measurements were from mid-foot to 90 cm (usually 80 cm) above the heel.

The reason that oedema measured from circumferences was less than from water displacement may have been because the circumferences were measured to 60 cm, or even 80 cm, from the heel and likely included in the measurements normal, or nearly normal, limb. The water method by contrast measured only the distal, more oedematous, region (30-50 cm from the heel).

This hypothesis was confirmed by using only the circumferences of the lower leg (mid-foot to 30 cm above the heel) for the 1,300 estimations of leg oedema. Now the regression equation for oedema estimated from the circumferences = 0.985 × (oedema by water displacement)-0.0089 (S.E.'s 0.037 and 0.002, respectively). The correlation coefficient altered little, being now 0.956.

A similar comparison of 150 unilateral lymphoedema arms (M. Mason, personal communication, 1989-1992) gave a correlation coefficient of 0.925 and a regression equation for oedema estimated from the circumferences = 1.096 × (oedema by water displacement)

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+0.007 (S.E.'s: 0.0567 and 0.006, respectively). These measurements were obtained by water displacement from the tips of the fingers to the top of the arm; circumferences were measured at mid-hand, the narrowest part of the wrist, and then at 10 cm intervals from the fingertips. Clinically, it was noted that the arms were more uniformly oedematous than the legs. The two methods thereby produce results which are nearly identical for statistical purposes of comparing one treatment with another. They differ only according to how much normal limb is included in the measurements.

In all measurements there is the problem of how much of the limb to incorporate into the determinations. If the oedema is largely distal and if one measures too far proximally, some component of normal limb is added to the oedematous part, thus "diluting" an alteration of the oedema. On the other hand, it is crucial to measure normal parts of a limb to ensure that physical treatment employed is not simply displacing the excess fluid into previously normal tissue regions and thus extending the oedema. (For example, we sometimes see genitalia, previously clinically normal, which has been made grossly oedematous by pneumatic pump compression and oedema displacement, and foolish modesty forbidding exposure of the genitalia and cessation of pneumatic compression.)

The best solution may be to measure all parts of the limb, but to report the results of only those portions which are oedematous. For this purpose, multiple measurements of circumference have greater flexibility than the fixed height to which the oedema is measured by water displacement, which once set at the initial measurement should not be altered later. Measurements of limb circumference at several points is of great value during treatment by Complex Physical Therapy. These measurements permit individual parts of the limb to be observed and the compressive technique to be modified accordingly. Practical aspects of such measurements have been discussed elsewhere (4).

When considering the actual amount of oedema (e.g., when modeling a whole lymphoedematous limb mathematically), one must recognize that most of the limb does not swell at all, or — if it does — only minimally. Thus, in the portion of the limb enclosed by the deep fascia, the increase in volume is far less than that of the epifascial zone (4,6). Hence oedema in the superficial compartment is actually much greater than that considered over the whole limb, and its assessment requires estimations of the relative volumes of both compartments.

To compare the amount of oedema, before and after treatment, does not require use of truncated cones. Simply adding the circumferences of various standardized parts of the limb and using this sum rather than the volume, before and after treatment, gives results very close to those obtained by calculating the volumes. Comparison of these two methods for 36 patients gave a correlation coefficient of 0.953. But this is not as exact.

Since varying degrees of exercise and the environment may markedly alter limb volume, it is usually assumed in unilateral lymphoedema that the contralateral normal limb should always be used as a control and measured as often as the abnormal one. Adjustments can be made if one or the other is the dominant arm, but the mean difference is only about 30 ml (5). Nonetheless, there are some inherent problems with use of a contralateral limb as the reference normal (see below).

PART 2 – REPRESENTING AND REPORTING OEDEMA: Equations Representing Alterations in Oedema

In Both Bilateral and Unilateral Oedemas

Oedema, and its alterations can be represented by a number of different equations. The simplest is just the final volume minus the initial one (the result is negative if there is a reduction):

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Difference in the Volume of the Limb = Final Volume-Initial Volume; i.e. $D=F-I$ eqn 1

Bodies and limbs, however, vary in size. This difference should be divided by some measure of the limb size. Even in bilateral oedema, the initial or final volumes of the affected limb can be used:

Difference in Volume/Initial Volume = $D/I=(F-I)/I=F/I-1$ eqn 2

Difference in Volume/Final Volume = $D/F=(F-I)/F=1-I/F$ eqn 3

Curiously, one might think that the final volume, being closer to the normal one, would be "better" (i.e eqn. 3 rather than eqn. 2). In fact, the reverse is true since errors in eqn. 3 are greater than those in eqn. 2 (see below). The right-hand sides of eqns. 2 to 7 can be multiplied by 100 to give percentages.

Unilateral Oedema

A contralateral limb can be a normal control to give initial and final amounts of oedema, relative to normal:

Initial Oedema = [Initial Volume-Normal Volume (at start)]/Normal Volume (at start);

$O_i=(I-N_i)/N_i=I/N_i-1$ eqn 4

Final Oedema = [Final Volume-Normal Volume (at end)]/Normal Volume (at end);

$O_f=(F-N_f)/N_f=F/N_f-1$ eqn 5

These values together give the difference in the amount of oedema, relative to the normal limb:

Difference in Oedema = Final Volume/Normal (at end)-Initial Volume/Normal (at start);

$O_d=O_f-O_i=F/N_f-I/N_i\approx(F-I)/N$ $\{if\ N_f\approx N_i\approx N\}$ eqn 6

The equation simplifies to its second form if the initial and final volumes of the normal limb are equal.

However what we, and indeed the patient, are ultimately interested in is the change in the amount of oedema, not relative to the normal limb, but relative to its initial amount:

Change in Oedema = Difference in Oedema/Initial Oedema;

$O_c=O_d/O_i=(F/N_f-I/N_i)/(I/N_i-1)\approx(F-I)/(I-N)$ $\{if\ N_f\approx N_i\approx N\}$ eqn 7

The second form again applies if the initial and final volumes of the normal limb are equal.

Eqn. 7 appears intuitively to be the most meaningful way of expressing an alteration in the amount of oedema, assuming that there is a contralateral normal or control limb. However this does not necessarily mean that the errors in this expression are least.

Effects of an Error in 'I', 'F' or 'N' in the Equations Representing Oedema

If an error (e) is made in one of the measurements (I, F, N_i or N_f), the equations become:

Difference in Volume of Limb: $D_e=F-I+e$ eqn 1e

Difference in Volume/Initial Volume: $(D/I)_e=(F+e)/I-1$ or $F/(I+e)-1$ eqn 2e

Difference in Volume/Final Volume: $(D/F)_e=1-(I+e)/F$ or $1-I/(F+e)$ eqn 3e

Initial Oedema; $O_{i,e}=(I+e)/N_i-1$ or $I/(N_i+e)-1$ eqn 4e

Final Oedema: $O_{f,e}=(F+e)/N_f-1$ or $F/(N_f+e)-1$ eqn 5e

Diff. in Oed.: $O_{d,e}=(F+e)/N_f-I/N_i$ or $F/N_f-(I+e)/N_i$ or $F/(N_f+e)-I/N_i$ or $F/N_f-I/(N_i+e)$
$\approx(F-I+e)/N$ or $(F-I)/(N+e)$ $\{if\ N_f\approx N_i\approx N\}$ eqn 6e

Change in Oed.: $O_{c,e}=[(F+e)/N_f-I/N_i]/(I/N_i-1)$ or $[F/N_f-(I+e)/N_i]/[(I+e)/N_i-1]$ or
$[F/(N_f+e)-I/N_i]/(I/N_i-1)$ or $[F/N_f-I/(N_i+e)]/[I/(N_i+e)-1]\approx[(F+e)-I]/(I-N)$ or $[F-(I+e)]/[(I+e)-N]$
or $(F-I)/[I-(N+e)]$ $\{if\ N_f\approx N_i\approx N\}$ eqn 7e

In all these equations (other than eqn. 1) there are as many alternative new error equations as there are variables, since an error in each of the variables produces different terms in the equation.

The important consideration is the relative error relative to its true value, i.e.:

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"D_e/D", "$(D/I)_e/(D/I)$", "$O_{d,e}/O_d$", etc. Using eqns. 1 and 2 as examples:

Difference in Volume of Limb: $D_e/D=(F-I+e)/(F-I)=1+e/(F-I)$ eqn 1p

Difference in Volume/Initial Volume: $(D/I)_e/(D/I)$

$=[(F+e)/I-1]/[F/I-1]=1+e/(F-I)$ or $[F/(I+e)-1]/[F/I - 1]$

$\approx 1-e\times F/[I^2\times(F/I-1)]=1-e\times F/[I\times(F-I)]$ {Because $e^2 << I^2$} eqn 2p

The method of simplifying the equations is shown in the second alternative of eqn. 2p, when the error was in I, *viz.* multiplying top and bottom lines by, here, (I-e); in the bottom line $e^2<<I^2$ and hence is negligible. Simplifications of all the proportional errors for all the equations are shown *(Table 1).*

For any given variable (I, F, or N), eqn. 7 gives a relative absolute error ≤ any of the others (including eqn. 6 when I>2N, but not when I<2N). Absolute errors in eqn. 2 are also often among the least and, in particular, are always ≤ that in eqn. 3 (Thus, F/I<I/F, if therapy is successful to any extent). Hence, eqns. 2, 6 or 7 are the best representation of alterations in oedema. Inspection also shows that errors in I usually have less effect than errors in F, which in turn usually have less effect than errors in N.

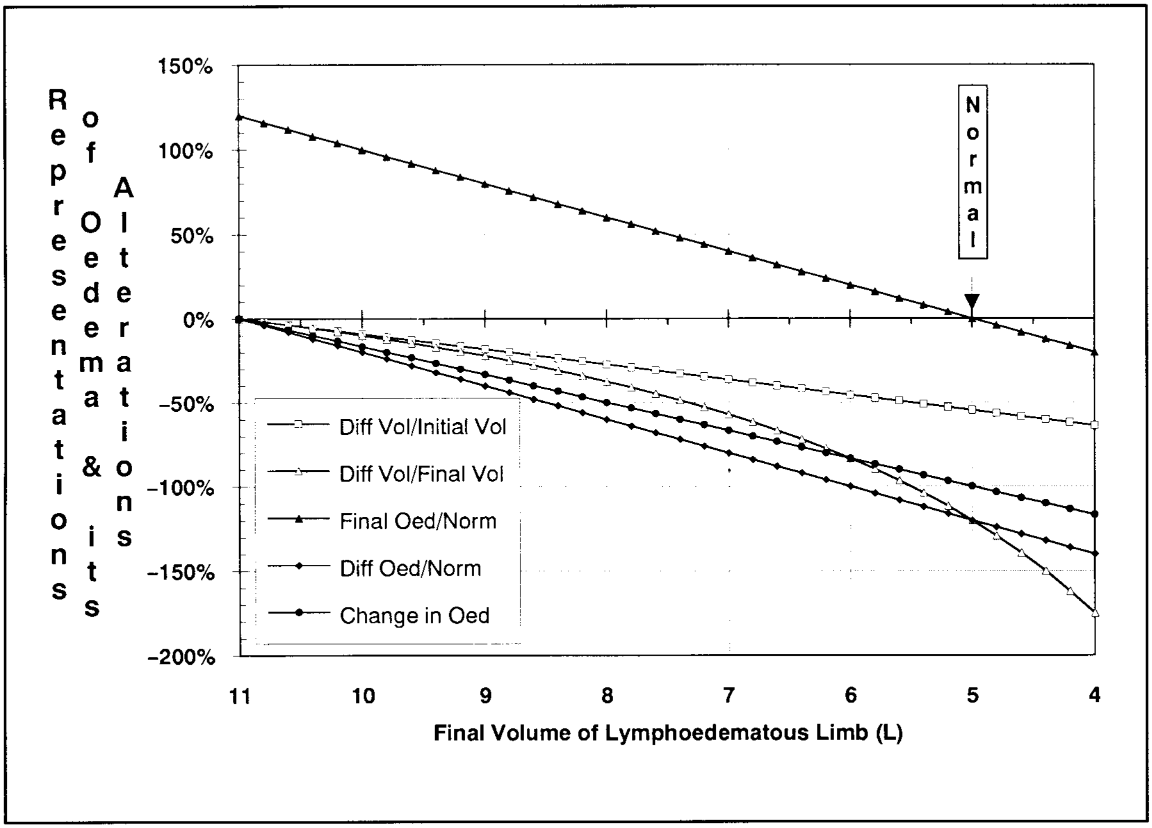

Fig. 1. A model of oedema with an initial volume of 11 L, a normal volume of 5 L and final volumes ranging from 11 to 4 L (corresponding to reductions of from 0 to -7 L). The size of the normal limb is shown. The legend shows the varying ways of representing oedema (omitting just the "Difference in Volume"). In all Figures, the plots are calculated values from the equations; markers are shown to identify the various lines. The value for "Initial Oedema/Normal" is where the "Final Oedema/Normal" meets the y-axis (at 120%). The latter then steadily reduces, crossing the x-axis when F=N. Most representations reduce linearly, contrasting with the non-linear reduction of "Difference in Volume/Final Volume". The "Change in Oedema" becomes -100% when F=N.

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Models of Errors in Oedema

The above only gives inequalities. Relative errors in eqns. 2, 6 or 7 are least, but the question is: "least by how much?" and "how often?" To answer this we need models of oedema, with introduced errors in the variables and calculations of the relative errors in the different representations of oedema and its alterations.

Fig. 1 shows a model of an oedema, with: I=11 L, N=5 L, reductions vary from 0 L to -7 L (i.e. F=11 L to 4 L). Nothing is altered if all variables are altered proportionally (e.g. I=5.5 L, N=2.5 L, and F=5.5 to 2 L). Representations of oedema are linear, except "Difference in Volume/Final Volume".

In *Figs. 2 to 4*, a 5% error has been added to I, F and N, respectively, and the relative percentage errors in the various representations of oedema and its alterations have been plotted against various values of F. If the error is -5%, the resultant graphs are approximately mirror images about the x-axes.

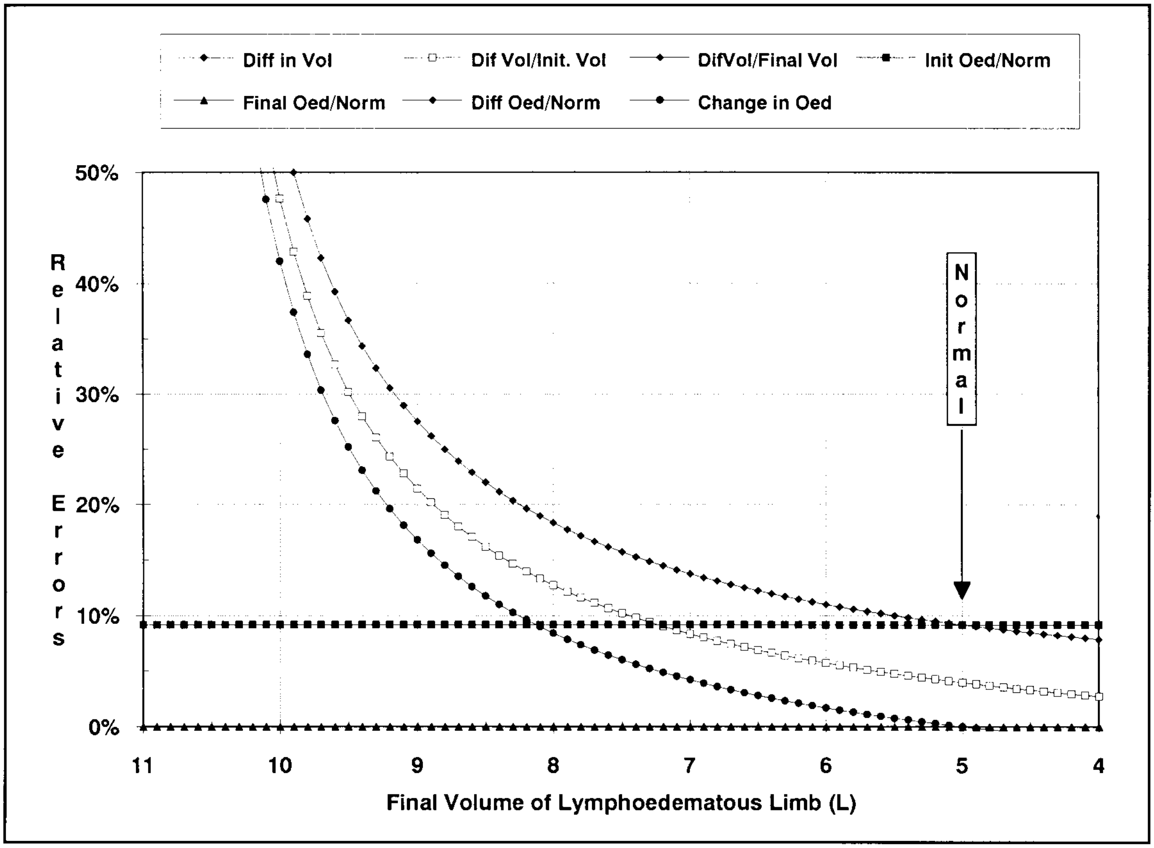

Fig. 2. The relative percentage errors in various representations of oedema for the model shown in Fig. 1, but with an error of 5% in the initial volume. (In this and the subsequent Figures, if the error is -5% the result is almost a mirror image of these graphs, symmetrical about the x-axis.) Errors in "Difference in Volume", "Difference in Volume/Final Volume" and "Difference in Oedema/Normal" are all identical (-♦-). (In these Figures, when graphs are identical some of the symbols for some of the graphs had to be altered since often only one is visible.) In this and Fig. 3, the error in "Difference in Volume/Final Volume" (here overlayed by -♦-) is always greater (absolutely) than "Difference in Volume/Initial Volume" (-❑-). Here and in Fig. 3 when F → I most of the errors become very great - including the "Difference in Oedema/Normal" even though this equals ("Initial Oedema/Normal" - "Final Oedema/Normal") both of which are constants in Fig. 2, and fairly constant in this range in Fig. 3!

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TABLE 1
Relative Errors in Representations of Oedema, According to the Variables

Eq		Error in I	Error in F	Error in N_i	Error in N_f
D	1	$e/(F-I)$	$e/(F-I)$		
D/I	2	$e \times F/[I \times (F-I)]$	$e/(F-I)$		
D/F	3	$e/(F-I)$	$e \times I/[F \times (F-I)]$		
O_i	4	$e/(I-N_i)$		$e \times I/[N_i \times (I-N_i)]$	
O_f	5		$e/(F-N_f)$		$e \times F/[N_f \times (F-N_f)]$
O_d	6	$e \times N_f/(F \times N_i - I \times N_f)$	$e \times N_i/(F \times N - I \times N_f)$	$e \times I \times N_f/[N_i \times (F \times N_i - I \times N_f)$	$e \times F \times N_i/[N_f \times (F \times N_i - I \times N_f)$
$O_d\{N\}$	6N	$e/(F-I)$	$e/(F-I)$	e/N	e/N
O_c	7	$e \times N_f/(F \times N_i - I \times N_f) + e/(I-N_i)$	$e \times N_i/(F \times N_i - I. \times N_f)$	$e \times I/N_i \times (I-N_i)] + e \times I \times N_f/[N_i(F \times N_i - I \times N_f)]$	$e \times F \times N_i/[N_f \times (F \times N_i - I \times N_f)$
$O_c\{N\}$	7N	$e/(F-I)+e/(I-N)$	$e/(F-I)$	$e/(I-N)$	$e/(I-N)$
Rel. Error		$7N \approx 7<2<1=3=6N$ $7 \,\&\, 2<4 (\text{as } F \to N)$	$2=1=6N=7N \approx$ $6=7<(u)5<(u)3$	$7<(u)6N<6<4>7N$ $7N<>6N\{\textit{if } I><2N\}$	$6N<6=7<5>7N$ $7N<>6N\{\textit{if } I><2N\}$

Numbers of the equations are shown in the second to left column. "6N" or "7N" are when $N_f \approx N_i \approx N$; these are from the simplified expression (in N) in eqns. 6e and 7e, not the originals involving N_i and N_f.

The final row compares the relative errors in the various equations, using the absolute values of the functions of (I,F,N_i,N_f), assuming $I>F>N$, and noting that $(F-I)<0$ while $(I-N)>0$. Some inequalities are only true if $F>N$; some are only usually true (u).

As $F \to I$, many relative errors become very large for errors in I or F *(Figs. 2 and 3)*, but not for errors in N *(Fig. 4)*. These include those in "Difference in Oedema/Normal" even though this equals ("Initial Oedema/Normal" – "Final Oedema/Normal") both of which are constants in *Fig. 2*, and fairly constant in this range in *Fig. 3*! Errors in either F or N cause the error in the "Final Oedema/Normal" to become undefined at about $F \approx N$ *(Figs. 3 and 4)*. Yet, "Difference in Oedema/Normal" (which equals this subtracted from the constant "Initial Oedema/Normal") is regular in this range *(Fig. 3)* or even constant *(Fig. 4)*! These surprises arise because the error terms sometimes cancel each other out in eqns. 4, 5 and 6.

Apart from equations in which an error can not arise because they do not contain the relevant term (plotted on the x-axis), the "Change in Oedema" (eqn. 7) has the least error of all representations of alterations of oedema (eqns. 1-3, 6 and 7). (This is true for errors in I and N - *Figs. 2 and 4*.) For errors in F *(Fig. 3)*, all are identical except "Difference in Volume/Final Volume" (eqn. 3), which is larger.

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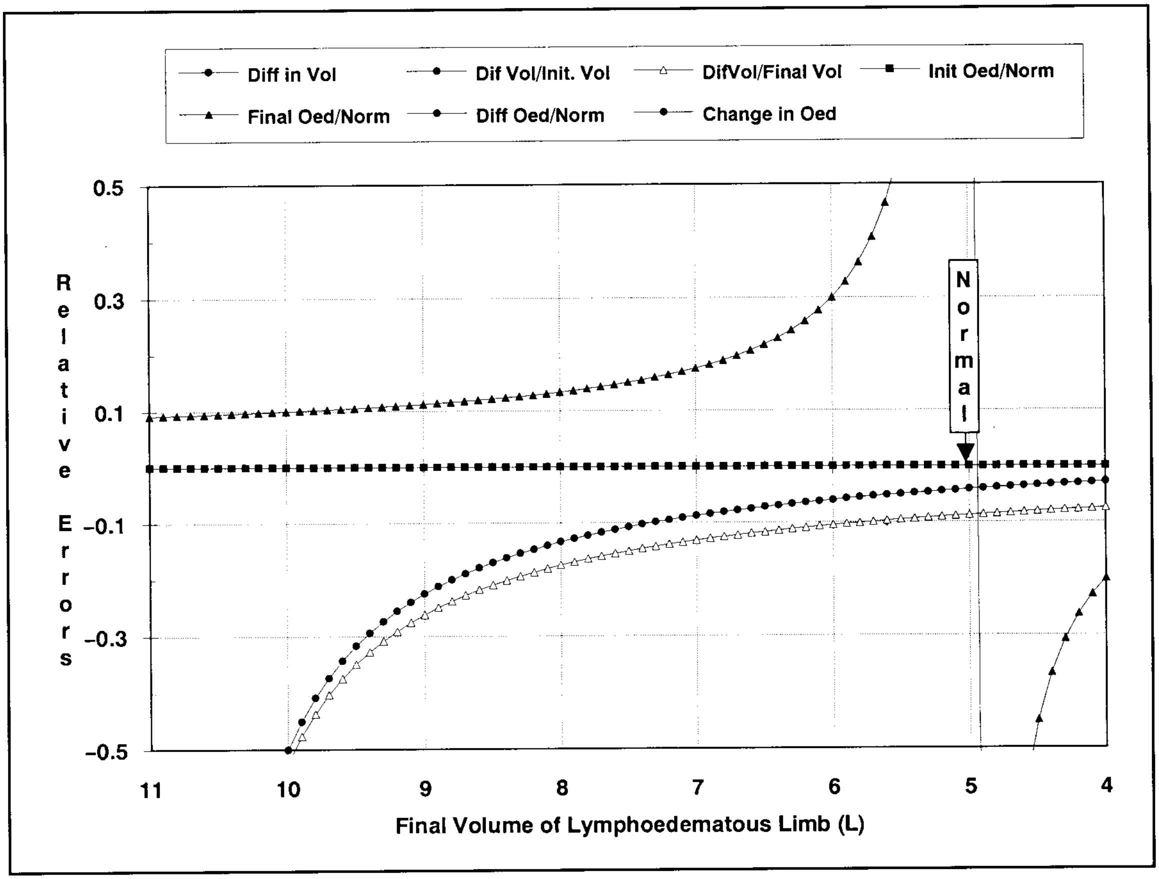

Fig. 3. As for Fig. 2, but with a 5% error in the final volume. The relative errors in "Difference in Volume", "Difference in Volume/Initial Volume", "Difference in Oedema/Normal" and "Change in Oedema" are all identical (-● -). That of "Final Oedema/Normal" (-▲-) is undefined at about the normal volume. However here and in Fig. 4 the error in "Difference in Oedema/Normal" (-● -) which equals this subtracted from the constant "Initial Oedema/Normal", is quite regular over this range.

Only eqns. 1-3 apply in bilateral oedema. For errors in I, "Difference in Volume/Initial Volume" (eqn. 2) always has the least error *(Fig. 2)*. For errors in F *(Fig. 3)*, the error in the "Difference in Volume" (eqn. 1) equals that in the "Difference in Volume/Initial Volume" (eqn. 2); both are less than that in the "Difference in Volume/Final Volume" (eqn. 3). Errors in N do not apply *(Fig. 4)*.

Variations in Error with the Amount of Oedema

The results so far have been from only one model of oedema, where the initial oedema is slightly more than the normal limb. However relative errors in the different equations vary with initial oedema. In *Table 2* three different oedemas are shown: severe (I>2N), moderate (I=1.6N), and mild (I=1.2N).

For an error in N, the error in "Difference in Oedema/Normal" (eqn. 6) is constant for all amounts of oedema, and "Change in Oedema" (eqn. 7) is less than this if I>2N, but it is greater if I<2N. On the other hand, eqn. 7 has an error which is always ≤ (often <<) all the

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TABLE 2
Variations in Percentage Relative Errors as Oedema Varies

Initial Volume	Final Volume	Normal Volume	(Final-Init)	(Final-Init) / Initial Vol	(Final-Init) / Final Vol	Init. Oed. / Normal	Final Oed. / Normal	(F-I)Oed / Normal	(F-I)Oed / Init.Oed.
Name of Representation			Dif.inVol	Dif.V./Init.V	Dif.V./Fin.V	Init.O./N	Fin.Oed./N	Dif.Oed/N	Change O.
Equation Number			1	2	3	4	5	6	7
Severe Lymphoedema, + 5% in Initial Volumes									
11,000	10,000	5,000	55%	48%	55%	9%		55%	42%
	8,000		18%	13%	18%	9%		18%	8.4%
	6,000		11%	5.7%	11%	9%		11%	1.7%
	5,000		9.2%	4.0%	9.2%	9%		9.2%	0.0%
	4,500		8.5%	3.3%	8.5%	9%		8.5%	-0.7%
+5% Error in Final Volumes									
11,000 ml	10,000	5,000	-50%	-50%	-53%		10%	-50%	-50%
	8,000		-13%	-13%	-17%		13%	-13%	-13%
	6,000		-6.0%	-6.0%	-10%		30%	-6.0%	-6.0%
	5,000		-4.2%	-4.2%	-8.7%		undefined	-4.2%	-4.2%
	4,500		-3.5%	-3.5%	-8.1%		-45%	-3.5%	-3.5%
+5% Error in Normal Volumes									
11,000	10,000	5,000				-9%	-10%	-4.8%	4.3%
	8,000					-9%	-13%	-4.8%	4.3%
	6,000					-9%	-29%	-4.8%	4.3%
	5,000					-9%	undefined	-4.8%	4.3%
	4,500					-9%	43%	-4.8%	4.3%
Moderate Lymphoedema, +5% Error in Initial Volumes									
8,000	7,500	5,000	80%	71%	80%	13%		80%	59%
	6,500		27%	21%	27%	13%		27%	12%
	5,750		18%	12%	18%	13%		18%	3.9%
	5,000		13%	8%	13%	13%		13%	0.0%
	4,500		11%	6%	11%	13%		11%	-1.7%
+5% Error in Final Volumes									
8,000	7,500	5,000	-75%	-75%	-76%		15%	-75%	-75%
	6,500		-22%	-22%	-25%		22%	-22%	-22%
	5,750		-13%	-13%	-17%		38%	-13%	-13%
	5,000		-8.3%	-8.3%	-13%		undefined	-8.3%	-8.3%
	4,500		-6.4%	-6.4%	-11%		-45%	-6.4%	-6.4%
+5% Error in Normal Volumes									
8,000	7,500	5,000				-13%	-14%	-4.8%	9.1%
	6,500					-13%	-21%	-4.8%	9.1%
	5,750					-13%	-37%	-4.8%	9.1%
	5,000					-13%	undefined	-4.8%	9.1%
	4,500					-13%	43%	-4.8%	9.1%
Mild Lymphoedema, +5% Error in Initial Volumes									
6,000	5,800	5,000	150%	138%	150%	30%		150%	92%
	5,500		60%	52%	60%	30%		60%	23%
	5,250		40%	33%	40%	30%		40%	7.7%
	5,000		30%	24%	30%	30%		30%	0.0%
	4,500		20%	14%	20%	30%		20%	-7.7%
+5% Error in Final Volumes									
6,000	5,800	5,000	-145%	-145%	-145%		36%	-145%	-145%
	5,500		-55%	-55%	-57%		55%	-55%	-55%
	5,250		-35%	-35%	-38%		105%	-35%	-35%
	5,000		-25%	-25%	-29%		undefined	-25%	-25%
	4,500		-15%	-15%	-19%		-45%	-15%	-15%
+5% Error in Normal Volumes									
6,000	5,800	5,000				-29%	-35%	-4.8%	33%
	5,500					-29%	-52%	-4.8%	33%
	5,250					-29%	-100%	-4.8%	33%
	5,000					-29%	undefined	-4.8%	33%
	4,500					-29%	43%	-4.8%	33%

Errors in I, F and N, were introduced into eqns. 1 to 7 and the errors in their results, relative to the true results, are shown for various values of I, F and N. Blanks indicate a term does not occur in the relevant equation, except in Columns 1 and 3 where blanks indicate no alteration from the value in the first row of that section.

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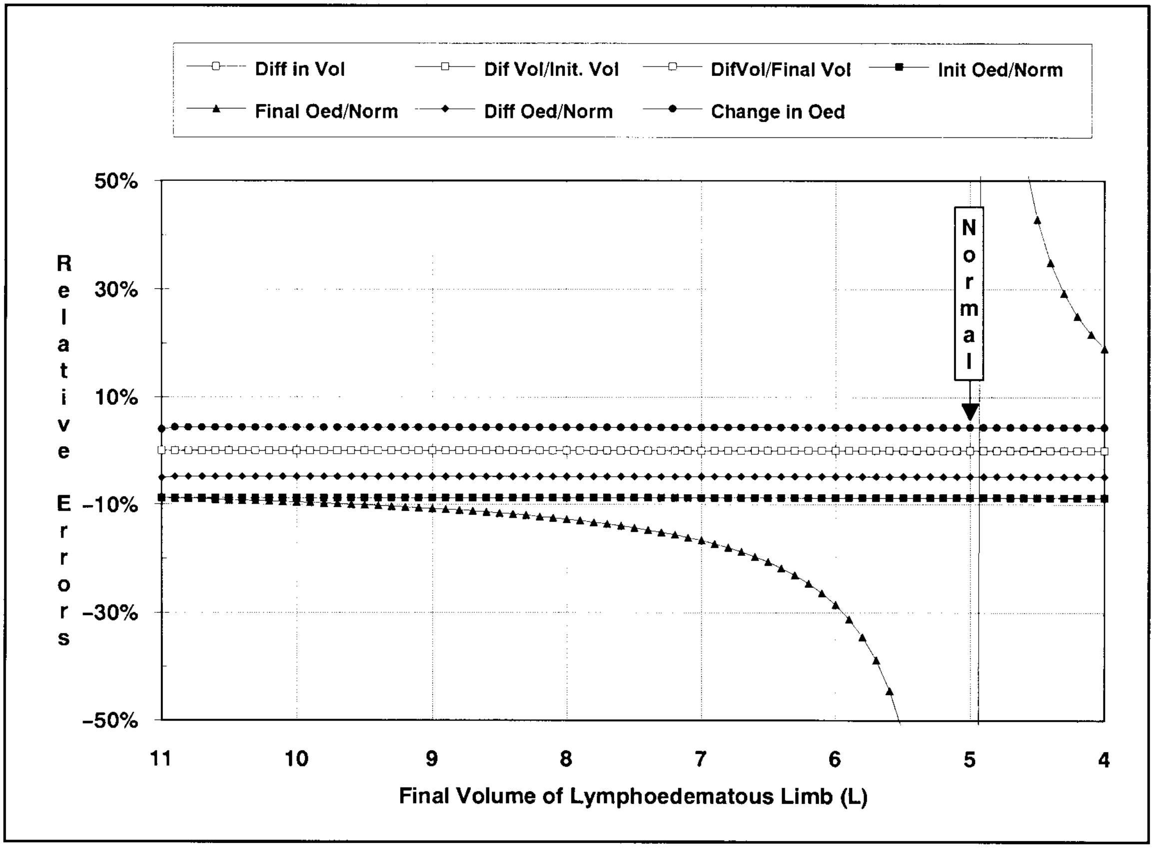

Fig. 4. As for Fig. 2, but with a 5% error in the normal volume. The relative errors in the "Difference in Volume", the "Difference in Volume/Initial Volume" and the "Difference in Volume/Final Volume" are identical (-❑-) and equal 0, since none contain "N".

errors in any other equation if the error is in I or F. Since errors in N will only occur at one half of the frequency of errors in I and F combined, and if I > 2N, it might be better to use eqn. 7 rather than eqn. 6.

However errors in "Difference in Volume/ Initial Volume" (eqn. 2) are < those in any except eqn. 7 for errors in I and = these for errors in F. None occur for errors in N. Hence eqn. 2 is very useful from the point of view of errors; but it does not relate the alteration in the oedema to normal. Eqns. 1 and 3 always have errors ≥ that from eqn. 2. So it is always better to use eqn. 2 than eqns. 1 or 3.

The same conclusions are reached if one integrates the error equations (1e to 7e) over the whole range, using I=11 L, F varies from 11 to 4 L and N varies from 11 to 4 L, assuming 5% errors in each variable in turn. (This is the model used to give *Figs. 5-8*, below.) The integrals of the error, over these ranges of F and N, for eqns. 1-7 are in the proportions: 1.2:1.1:1.2:0.9:11:1.1:1.0, respectively. Thus the total errors in eqn. 7 < eqn. 6=eqn. 2<eqn. 1=eqn. 3. However the differences are small when considered over the whole ranges of F and N.

Distortions Caused By The Non-Linearity of Some Equations

There are, however, other considerations than just the possibility of errors in measurements. These are the effects of non-

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TABLE 3
Correlation Coefficients of Various Representations of Oedema

	Eqn	Dif. in Vol 1		Dif.V/Int.V 2		Init.Oed/N 4		Fin.Oed/N 5		Dif.Oed/N 6		Change Oed 7	
Dif.Vol/Init.V1	2	.531	***										
Init.Oed/Nm	4	-.452	***	-.621	***								
Final Oed/Nm	5	-.134	NS	-.022	NS	.782	***						
Diff.Oed/Nm	6	.573	***	.954	***	-.765	***	-.196	**				
Change Oed	7	.057	NS	.184	**	.297	***	.527	***	.078	NS		
-log(-Change in Oedema)	-l0g(-7)	.164	*	.379	***	.201	**	.553	***	.256	***	.833	***

Based on 200 patients with unilateral lymphoedema of the arm or leg, all treated with Complex Physical Therapy and some with benzo-pyrones, at The Adelaide Lymphoedema Clinic and The Sydney Lymphoedema Practice.

Equations 4 and 5 represent amounts of oedema rather than alterations in it.

The significances refer to the correlation coefficient on their left: "NS" signifies p>0.05, * signifies 0.05>p>0.01, ** signifies 0.01>p>0.001, *** signifies 0.001 > p.

linearity in some of the equations with respect to the variables F and N. (Although eqns. 2 and 7 are non-linear in I, this is not considered here since I is regarded as a constant.)

These non-linearities cause some representations of oedema to have little correlation with others *(Table 3)*. While the significances of these correlations are often very large (but by no means always), the actual coefficients are surprisingly poor. One representation of oedema, or of its alteration, often yields results which are very different from another. Close correlation is only shown by eqns. 2 and 6 ("Difference in Volume/ Initial Volume" and "Difference of Oedema/Normal"), followed by eqn. 4 with eqns. 5 or 6 ("Initial Oedema/Normal" with "Final Oedema/Normal" or "Difference of Oedema/Normal"). Yet eqns. 5 and 6 have very little correlation with each other!

Because of the great non-linearity of eqn. 7 ("Change in Oedema"-*see below*), its results were made more linear (final Row of *Table 3*) by using: - $\log_{10}$(-"Change in Oedema"). In fact, however, this achieved very little—as can be seen by comparing the coefficients in the last two Rows *(Table 3)*.

The lack of correlation between the equations comes from the way in which the results of some of them vary disproportionally as F and N vary. From eqns. 1 and 2, it can be seen that an alteration in F causes a constant proportional change in D and D/I. However eqn. 3 is non-linear in F. As $F \rightarrow I$, a constant change in F causes a much smaller alteration in I/F. Similarly eqns. 4 and 5 are non-linear in N. However the shapes of their plots are far from identical *(Figs. 5 vs. 6)*, since F does not occur in eqn. 4 and does in eqn. 5 (although this equation is linear in F for a given N - *Fig. 6*).

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Eqn. 6 is also linear in F for a given N, but non-linear in N *(Fig. 7)*. The convexity of eqn. 6 is opposite to that of eqn. 5 and the two equations are most non-linear at the opposite ends of the range of F (Because eqn. 6=eqn. 4-eqn. 5.)

Eqn. 7 is also linear in F for a given N, but very non-linear in N, especially as $N \rightarrow I$ *(Fig. 8)*. Its convexity is opposite to those of all eqns. 4 to 6. In particular, not only is the non-linearity of eqn. 7 opposite to that of eqn. 6, but the non-linearities are greatest at the opposite ends of the range of N, and the convexity in the plot of eqn. 7 is much greater than that in the plot of eqn. 6.

Integrating the absolute values of the various equations over the whole ranges of F and N in this model, gives the volumes between 0 and the results of eqns. 2-7. They are in the proportions 1.0:1.1:1.5:5.3:1.0:602, respectively, when compared with their values for some arbitrary standard (F=6 and N=5). The non-linearity of eqn. 7 clearly has very large effects.

Thus all of these equations (especially eqn. 7) can give markedly differing results for oedema and its alterations. It is important to realize that these differences do not necessarily imply that one equation is correct and the rest are wrong; it simply means that they measure different things. On the other hand, some of these things may be more meaningful and useful than others.

Coping with Abnormal "Normal" Limbs

Consider an oedema which is changed to -400% of its initial amount (using eqn. 7) (e.g., I=6, F=4 and N=5.5; this is not purely hypothetical, but has happened with several of our patients). The most likely reason for this finding is that, while each measurement may have been made with perfect accuracy, the normal volume of the affected limb is simply much less than that of the unaffected contralateral one. This discrepancy may be due to muscle wasting or to being the non-dominant side (6). Evidently here N is not equal to the other limb, but to some (unknown) lesser volume. Non-linearity in eqn. 7 makes this effect greater as $N \rightarrow I$ and as $F \rightarrow << N$. The "Change in Oedema" is certainly in error, not an error caused by inaccurate measurement, but one caused by not knowing what N really is and the assumption that the body is symmetrical. This factor also causes other errors in statistics using this value because the frequency distribution of the results is skewed. This distortion can in principle be normalized, e.g. by using logarithms, but the last Row of *Table 3* shows that this attempt at normalization is not always effective.

Eqns. 2 and 6 ("Difference in Volume/Initial Volume" and "Difference of Oedema/Normal") are very highly correlated *(Table 3)*. Does this mean that these equations are the best to use, and eqn. 2 is ideal because it applies to both unilateral and bilateral oedema? Yet usage of eqn. 2 abandons all considerations of the size of the contralateral normal limb, which is a useful control for variations caused by the weather and exercise. It also dismisses consideration of the alteration in the amount of oedema itself, which is what chiefly concerns both patient and therapist!

Another possibility is to report the results of a number of equations (e.g. 2, 6 and 7). This adds complexity to a presentation and still does not solve the problem of the individual patient.

It is still possible to retain the normal limb as a control if its volume is used to modify the final measured volume. Thus, if hot weather caused the contralateral normal limb to swell, it might be assumed (although unproven) that the affected one swells in the same proportion. Its volume could be adjusted from the initial and final volumes of the normal limb. Eqns. 1 and 2 then become:

Difference in Volume of Limb:

$$D = F \times N_i / N_f - I \qquad \text{eqn 1m}$$

Difference in Volume/Initial Volume =

$$D/I = (F \times N_i / N_f - I)/I = F \times N_i / I \times N_f - 1, \qquad \text{eqn 2m}$$

Eqns. 4 -7 would be unchanged because they already take account of such relative alterations.

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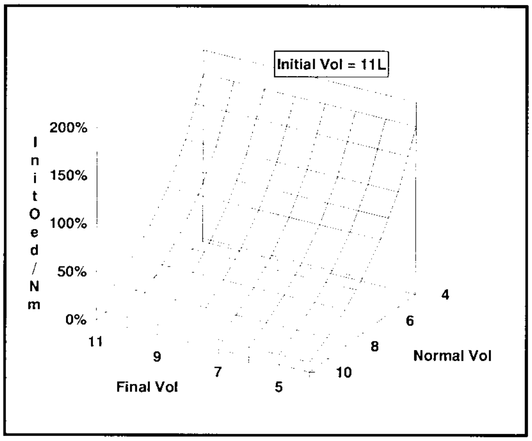

Fig. 5. The values of the "Initial Oedema/Normal" (eqn. 4) are plotted against various values of both F and N. As N → 4 L, small changes in the N cause greater changes in "Initial Oedema/Normal".

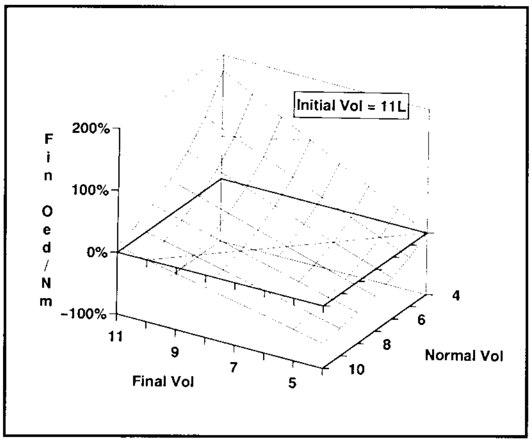

Fig. 6. As for Fig. 5, but the "Final Oedema/Normal" (eqn. 5) is plotted. Again as N → 4 L, small changes in N cause greater changes in "Final Oedema/Normal", but less severe ones.

It could be argued that perhaps even if the other limb is oedematous, but untreated, it could still be used to give a proportional adjustment to F. However this attitude assumes that treatment does not affect the contralateral limb. Yet, when performing Complex Physical Therapy (CPT) it is mandatory to look for changes in the opposite limb being treated, e.g., an enlargement from oedema being diverted from the oedematous limb to the other side with overloading of its lymphatics (4) (M Földi, personal communication, 1990). In primary or filaritic lymphoedema, or lymphedema after treatment of pelvic neoplasms, a clinically appearing normal leg often has abnormal lymphatic drainage and preclinical lymphoedema which is particularly vulnerable to such overloads. A similar sequence may occur in bilateral mastectomy with one arm appearing clinically "normal". On the other hand, CPT (which includes massage of the body quadrant adjacent to such a "normal" limb) sometimes results in the reduction of the "normal" limb volume as well as the grossly oedematous one, suggesting that subclinical lymphoedema was present in the normal appearing side.

Multiple circumference measurements at various distances show that such induced alterations in the "normal" limb initially occur proximally rather than distally (4). Accordingly, it might be possible to recognize or even adjust for these artifacts. However, it would be necessary to establish in each instance that the contralateral control limb was not altered by treatment. This requirement is far from a satisfactory arrangement.

APPLYING THIS IN PRACTICE

Unilateral Oedema—Individual Patients

Even a poor control is probably better than no control. A patient with unilateral oedema and the therapist both wish to know approximately how much oedema has been removed. A patient is more likely to understand "percentage of oedema" (eqn. 7) than "altered percentage of normal" (eqn. 6) and certainly more than "altered percentage of the initial volume" (eqn. 2). Eqns. 6 and 7 can be used (eqn. 7 unless F<N or N→I, when eqn. 6 is necessary). But eqn. 7 should be used with the proviso that it may be subject to unknown errors. Thus, if N has been altered by 5% because of therapy, the results of eqns. 6 or 7 are just as wrong as if its measurements were in error by 5% *(as in Table 2)*!

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CONCLUSIONS

In bilateral oedema it is best to use:

Difference in Volume/Initial Volume = D/I=(F-I)/I=F/I-1 eqn 2

In unilateral oedema it is best to use:

"Difference in Oedema" = Final/Normal (at end)-Initial/Normal (at start);

$O_d=O_f-O_i=F/N_f-I/N_i\approx(F-I)/N$ {*if* $N_f\approx N_i\approx N$} eqn 6

While possible as a rough estimate for an individual patient, it is better not to use \

"Change in Oedema" = Difference in Oedema/Initial Oedema;

$O_c=O_d/O_i=(F/N_f-I/N_i)/(I/N_i-1)\approx(F-I)/(I-N)$ {*if* $N_f\approx N_i\approx N$}, eqn 7

When presenting a series of patients, the mean of individual eqn. 7's should not be used, but a mean of eqn. 7 is reasonably accurate if it is obtained from the Means of eqns. 4 and 6, i.e.:

Mean (O_c)=Mean (O_d)/Mean (O_i), with O_i given by: $O_i=(I-N_i)/N_i=I/N_i-1$ eqn 4

If n > about 25, its S.E. is given by:

SE (O_c)={SE(O_d)^2+[Mean(O_d)^2 x SE(O_i)^2 /Mean(O_i)^2]}$^{1/2}$ /Mean(O_i) eqn 7s

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