Adjusting Cattle Numbers to Fluctuating Forage Production with Statistical Decision Theory

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Highlight

Statistical decision theory offers northern Nevada cattle producers an opportunity to increase their income by aiding in making adjustments in livestock numbers according to expected forage production. It is necessary for cattlemen to determine the number of cattle to carry through the winter before knowledge of forage supply for the coming year is available. Statistical decision theory provides a simple tool whereby ranchers may use observed information on precipitation to select the appropriate number of cattle to be wintered. Ninety-five years of weather data were used to evaluate this technique under ranch conditions in northern Nevada. Results indicate that statistical decision theory offers promise as a technique for maximizing the long-run average income of ranchers while making provision for protection of the range resource.

Como Ajustar el Numero de Animales a las Fluctuaciones en la Produccion Forrajera—una Aplicacion de la Teoria de Decision Estadistica

Resumen

Para ejemplificar este modelo bajo probabilidades a priori y a posteriori, se empleó una operación combinada de pié de cría (venta de crías al destete) y de pié de cría—engorda (venta de crías como añojos). Esta organización tiene muy buena flexibilidad para ajustarse a niveles de producción forrajera tanto abajo como arriba de lo normal. Por lo tanto, el valor del modelo expuesto fué más bien limitado para este sistema. El modelo resultó de mayor valor bajo el sistema menos flexible de una operación pié de cría.

Aunque los aumentos en los ingresos fueron modestos con el modelo, la técnica ofrece ciertas promesas para trabajar en este campo. La técnica puede aparecer complicada al principio, pero es más bien simple. Los resultados pueden presentarse en una forma sencilla, como aparecen en el cuadro 6, para aquellos no interesados en profundizarse en suposiciones y mayores datos.

Este modelo fué desarrollado para ayudar a los ganaderos a utilizar los reducidos datos disponibles en el otoño con el fin de predecir la cantidad de forraje disponible para el año siguiente. El modelo está planeado simplemente para tratar de obtener los máximos ingresos a largo plazo. Podría modificarse fácilmente para reflejar otras cosas tales como diferentes grados de utilización de forraje aceptable, niveles de ingreso mímimo anual, ó cualquier otra medida de importancia en el manejo del recurso pastizal. Esta es otra herramienta que puede ayudar al ganadero a aumentar sus ingresos y a la vez sirve para mejorar la utilización de un pastizal.

Cattlemen and range scientists are continually faced with the problem of adjusting stocking rates to fluctuating forage production. Currie and Peterson (1966), Houston and Woodward (1966), Reed and Peterson (1961), and several others have published research on this problem. Numerous "rules-of-thumb" have been advanced to aid decision makers in resolving this problem. Stoddart and Smith (1943) mention several in their text. Recent developments in the field of statistical decision theory may offer some new insights into this traditional problem. This study was designed to examine the usefulness of statistical decision theory in specifying adjustments in response to an uncertain forage supply in the sagebrush–grass range area of northern Nevada.

Annual fluctuations in forage production in this area are due primarily to fluctuations in precipitation. If future precipitation were known with certainty, it would be a simple matter for producers to adjust their operation to future forage supply. However, future precipitation is rather uncertain. Statistical decision theory was employed in an attempt to develop a procedure utilizing observable precipitation to predict future precipitation. From the precipitation prediction future forage production may be estimated, thereby facilitating adjustment of livestock numbers. The model developed in this study maximizes the weighted average of possible incomes for ranchers. A restraint preventing range use in excess of current year's production was imposed to prevent deterioration of the range.

Method of Analysis

The formal theory used was Bayesian statistics. A nontechnical presentation of the method may be found in Chernoff and Moses (1959). The more mathematically inclined may prefer works by Luce and Raiffa (1957) or Schlaifer (1959).

A ranch situation capable of running 400 cows under a conventional cow–calf system in normal years was synthesized to represent a typical set of feed resources found in northern Nevada. Three basic livestock systems were fitted to this feed base: (1) a strict cow–calf system, (2) a combination cow–calf and cow–yearling system, and (3) a strict cow–yearling system. Cow numbers were adjusted under each system so that production and consumption of forage would be equal for a "normal" year.

Monthly precipitation data from the Elko, Ne-

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Table 1. Frequencies of three precipitation levels.

Level	Amount (inches)	Years observed	*A priori* probability
Below normal	0 to 5.99	21	.221
Normal	6.0 to 10.99	57	.600
Above normal	11 or more	17	.179

vada weather station were obtained for 95 years (1870–1965). Three levels of precipitation were established. Median annual precipitation was 8.5 inches and served as the base for establishing the average feed resource for the ranch. A "below normal" level was established at 5 inches and an "above normal" level at 12 inches of precipitation. Frequencies for these precipitations are shown in Table 1. The forage production equation developed by Sneva and Hyder (1962) was used to estimate forage under the "below" and "above" normal years.[2]

Although ranchers have selected one of the basic livestock systems (i.e., cow–calf, combination, or cow–yearling), additional adjustments may be made within each system as forage supply fluctuates. Four such alternatives were assumed to exist: (1) sell all calves, (2) sell one-half and retain one-half of the calves, (3) keep all the calves, and (4) sell some of the cow-herd in addition to selling all the calves. Selection of an alternative depends upon the rancher's expectation of precipitation for the coming year. This decision must be made in the fall when annual precipitation is still an unknown.

The analysis to this point includes three possible precipitation levels, three livestock systems, and four possible actions within each system. This yields 36 possible outcomes. Income data from research by Peacock (1967) were used in evaluating the usefulness of this technique. In order to simplify the presentation, income data are shown only

[2] This forage estimating equation has not been validated for northern Nevada conditions. Therefore, forage yields for "above" and "below" normal precipitation years may be subject to some error. Readers are cautioned to view this work only as a demonstration of the application of the technique.

Table 2. Net ranch income for combination livestock system under three precipitation levels and four actions.

Action	Level: Below normal	Normal	Above normal
Sell all calves	– $5,324	$4,626	$ 7,102
Sell one-half calves	– 5,516	8,242	10,128
Keep all calves	– 6,260	6,115	14,482
Sell-down cows	– 1,614	1,159	3,145

for a 362 cow combination cow–calf and cow–yearling system (Table 2).

The 95 years of weather data were examined to determine the relationship between observed July through October precipitation and total precipitation for the year. It is necessary to develop *a posteriori* probabilities which are the conditional probabilities of the three levels of precipitation given that a particular level of July through October precipitation is observed on November 1. Table 3 illustrates the derivation of these *a posteriori* probabilities by Bayes' formula,

$$P(\Theta/Z) = \frac{P(\Theta)\,P(Z/\Theta)}{P(Z)}.$$

Detailed procedures for calculation of these probabilities may be found in Luce and Raiffa (1967). The following gives a brief explanation of the calculations appearing in Table 3. For example, the conditional probability figures appearing in the "below normal" precipitation year of Section I were calculated in the following manner. Below normal precipitation occurred in 21 of the 95 years of recorded weather data. In 17 of these 21 years less than one inch of precipitation was observed by November 1 (Z_1), while in the remaining 4 years between 1 and 1.99 inches were observed by November 1 (Z_2). Therefore given that a below normal precipitation year actually occurred, the conditional probability of observing less than one inch of precipitation by November 1 was .810, that of observing between 1.0 and 1.99 inches was .190, and there was zero probability of observing 2 or more inches of precipitation. Con-

Table 3. Determination of *a posteriori* probabilities of total precipitation based on observed precipitation, November 1.

Total precip. (Θ)	I. Conditional probabilities P(Z/Θ) — Observed precip., Nov. 1 (in.): 0–.99	1–1.99	2–2.99	3–3.99	Over 4	*A priori* probabilities P(Θ)	II. Joint probabilities P(Θ) P(Z/Θ) — Observed precip., Nov. 1 (in.): 0–.99	1–1.99	2–2.99	3–3.99	Over 4	III. *A posteriori* probabilities $P(\Theta/Z) = \frac{P(\Theta)\,P(Z/\Theta)}{P(Z)}$ — Observed precip., Nov. 1 (in.): 0–.99	1–1.99	2–2.99	3–3.99	Over 4
Below normal	.810	.190				.221	.1790	.0420				.4720	.1288			
Normal	.316	.368	.228	.053	.035	.600	.1896	.2208	.1368	.0318	.0210	.500	.6733	.7647	.6011	.3328
Above normal	.059	.353	.235	.118	.235	.179	.0106	.0632	.0421	.0211	.0421	.0280	.1939	.2353	.3989	.6672
						P(Z)	.3792	.3260	.1789	.0529	.0631					

Table 4. Expected value of action for combination system with 2 to 2.99 inches of observed precipitation.

Precipitation	Net ranch income: Sell all calves	Sell one-half calves	Keep all calves	Sell down cows	*A posteriori* probabilities
Below normal	–$5,324	–$5,516	–$6,260	–$1,614	.0000
Normal	4,626	8,242	6,115	1,159	.7647
Above normal	7,102	10,128	14,482	3,045	.2353
Expected value	$5,209	$8,686	$8,084	$1,603	1.0000

ditional probabilities for normal and above normal precipitation levels were derived similarly. The *a priori* probabilities shown in Table 3 are merely the percentage of years that each precipitation level occurred over the 95-year period.

The joint probabilities shown in Section II of Table 3 are simply an intermediate step needed to derive the *a posteriori* probabilities. These joint probabilities are the product of two distributions. The conditional probabilities shown in Section I are multiplied by the values appearing in the vector of *a priori* probabilities. As an example, the figure .1790 appearing in the upper left-hand corner of the joint probabilities section was obtained as the product: .810 × .221 = .1790. The P(Z) row is simply the sum of values appearing in the columns of the joint probabilities section.

The *a posteriori* probabilities are then derived according to Bayes' formula as shown in Section III of Table 3. This involves dividing each element in the joint probability matrix by the corresponding value in the P(Z) row. As an example, the figure .4720 appearing in the upper left-hand corner of Section III was obtained as the quotient of the fraction, .1790/.3792.

The *a posteriori* probabilities are then used by the decision maker in determining which action will maximize his weighted average net ranch income. A simplified example is shown in Table 4 considering only the 2 to 2.99 inch observed precipitation column. Income figures from Table 2 are multiplied by the respective *a posteriori* probability value (Section III, Table 3) for that precipitation level. As an example, $5,209 under "sell all calves" (Table 4) was calculated as: (–5,324) (0) + (4,626) (.7647) + (7,102) (.2353) = 5,209.

It can be seen from Table 4 that a rancher observing between 2 and 2.99 inches of rainfall by November 1 should decide to sell one-half of his calves and winter over the other one-half if he wishes to maximize his long-run average profit. Table 4 is easily expanded to calculate expected values for the other observed levels of precipitation.

Results

The payoff matrix for the combination livestock system using only *a priori* or long-run averages is shown in Table 5. As would be expected, the profit maximizing strategy, yielding $5,539 net ranch income, is indicated under sell one-half the calves and winter over one-half the calves.

The question of concern is whether the average income level obtained with use of only *a priori* probabilities can be increased with the decision model. If so, the rancher will have an increased income and the range will benefit from less overgrazing in low precipitation years. The decision theory payoff matrix using *a posteriori* probabilities is shown in Table 6.

Table 6. Net ranch incomes (dollars) with *a posteriori* probabilities.

Observed precip. Nov. 1 (inches)	Net ranch income ($): Sell all calves	Sell one-half calves	Keep all calves	Sell down cows
0 to 0.99	1	1,801	508	97
1 to 1.99	3,824	6,836	6,144	1,167
2 to 2.99	5,209	8,686	8,084	1,603
3 to 3.99	5,614	8,994	9,453	1,912
4 or more	6,278	9,500	11,697	2,418

Table 5. Net ranch incomes with *a priori* probabilities.

Precipitation	Net ranch income: Sell all calves	Sell one-half calves	Keep all calves	Sell down cows	*A priori* probabilities
Below normal	–$5,324	–$5,516	–$6,260	–$1,614	.221
Normal	4,624	8,242	6,115	1,159	.600
Above normal	7,102	10,128	14,482	3,045	.179
Expected income	$2,869	$5,539	$4,878	$ 883	1.000

The underlined figures indicate the correct action for each observed level of July through October precipitation. A mixed strategy is indicated. The basic action selling one-half and keeping one-half the calves is indicated for November 1 precipitation levels up to three inches. If three or more inches of precipitation are observed by November 1, the rancher should keep all his calves over the winter. The long-run expected value of this strategy is $5,704, an average increase of $162 for a ranch running approximately 362 cows. This is obtained by multiplying the underlined values by the appropriate P(Z) value shown as the bottom line in Section II of Table 3: i.e., (1,801) (.3792) + (6,836) (.3260) + (8,686) (.1789) + (9,453) (.0529) + (11,697) (.0631) = 5,704. The same model applied to the basic cow–calf system increased annual income $286.

Conclusions

A combination cow–calf and cow–yearling system was indicated using both the *a priori* and *a posteriori* probabilities. Such a combination livestock system has considerable inherent flexibility for adjustment to both below and above normal forage supplies. Therefore, increases in income with use of statistical decision theory were rather small for the combination cow–calf and cow–yearling system. The model was of greater value under the less flexible cow–calf system.

Although increases in income were modest, the technique offers some promise as a method for making decisions concerning adjustments of cattle numbers. The procedure as presented in this article may appear complicated at first exposure but is rather easily developed when broken down into separate steps. Results applicable for a particular area could be presented in a simple form (e.g., such as Table 6) for those not interested in the underlying assumptions and data.

This research was conducted to determine whether statistical decision theory might be useful in assisting ranchers to use precipitation data available to them in the fall for predicting forage for the following year. The model sought simply to maximize long-run expected income. It could be easily modified to reflect such things as different rates of acceptable forage utilization, minimum annual income levels, or other measures of importance in managing the range resource. Statistical decision theory, employing Bayesian statistics, is another tool which may help increase rancher income while serving to improve range utilization.

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A. W. Sampson Portrait

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