A Dynamic Programming Application for Short-term Grazing Management Decisions

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Abstract

This study emphasizes short-term management decisions that are made in a yearling cattle operation in northeastern Colorado. Empirical equations describing forage and animal growth were coupled with marketing and supplementation alternatives. Four cases were modeled with high and low stocking densities and partial or total sales strategies. Net present value of yearling steer sales were maximized using dynamic programming.

Early sale of cattle was an economically favorable alternative because of decreasing daily gains toward the end of the grazing season (September-October) and decreasing steer prices. Supplementation during September-October was also profitable to offset the decreasing trend in average daily gain caused by declining forage quality. Under the high stocking density and partial sales strategy, early sales regulated standing crop left at the end of the grazing season. Under the low stocking density and partial sales strategy, early sales partially offset net return losses for those animals that had to be sold at the traditional marketing date. The total sales strategy favored sales of livestock 2 weeks before traditional marketing under low and high stocking density and partial sales strategy. Net present values per pasture were slightly larger for the total sales strategy than the partial sales strategy using both low and high stocking densities.

Key Words: yearling grazing, supplementation and marketing alternatives, optimal control application, net present value maximization

Maximization of net returns or minimization of costs are major concerns in ranch management. In addition, considerations about cattle markets must be integrated into the net return equation. Accurate short term management decisions yield efficient forage utilization, high animal performance and profit maximization. Historically, most of the research done in range science decision models have dealt with long-term planning (Sharp 1967, Burt 1971, and Fisher 1985, among others) while short-term planning has been neglected (some exceptions are Bartlett et al. 1974, Hunter et al. 1976, Toff and O’Hanlon 1979).

This study emphasized the role of short-term decisions in forage and livestock management. The types of operation and management decision system employed were selected for their respective advantages. A yearling operation at the Central Plains Experimental Range (CPER) was chosen for study because yearling operations are more simply modeled and more sensitive to fluctuations in climate, forage quality, and market conditions than cow/calf operations. Dynamic programming (DP) was chosen over linear programming (LP) for various reasons:

1. The optimization problem can be defined in a free format (in contrast to matrix formulation needed for LP).
2. DP optimization can model nonlinear relations and does not require continuous functions to represent treatments.
3. While LP formulations increase in size to accommodate piece-wise approximations of non-linear constraints, DP addresses such constraints directly.

4. While multiperiod LP formulations require specifications of all possible values of the state variables for every period, DP narrows the solution space to specific values of state variables for different periods.
5. DP models may vary state variable growth rates. These rates can be dependent upon previous state variable values, state variable interactions, and time. In contrast, serial LP models, such as the ones developed by previous authors (Sharp 1967, Bartlett et al. 1974, Hunter et al. 1976, and Propoi 1979), show production rates which vary only with respect to time.

Dynamic optimization models yield a sequence of optimal decision rules useful to decision makers (i.e., range managers). These optimal sequential rules make it possible to analyze marginal changes in biological or economic aspects of the cattle operation (van Poollen and Leung 1986).

Dynamic programming, an optimal control method, is not used extensively in range science because the implementation of optimal control theory and the algorithms to solve optimal control problems are not well developed or widely understood (Trapp and Walker 1986). This paper will pursue a simple example of possible uses of deterministic DP in a yearling cattle operation.

Materials and Methods

Data Sources

This study utilized data from the Central Plains Experimental Range (CPER) as input for the dynamic optimization model. CPER is located in northeastern Colorado as part of the shortgrass prairie of the Central Great Plains. Rolling hills separated by wide swales constitute its topography, and the soils include patches of loamy clay and loamy sand soils. Mean annual precipitation is 311 mm (70% of this falls between April and September), with annual extremes from 110 to 510 mm (Sala et al. 1981). The vegetation is dominated by blue grama (Bouteloua gracilis (H.B.K.) Lag.) and buffalo grass (Buchloe dactyloides (Nutt) D.C.), constituting 65 to 90% of the forage grazed by cattle (Klipple and Costello 1960).

CPER’s yearling operation is typical of those in the Central Great Plains. Animals weighing 225 to 275 kg are bought in the spring, and are sold weighing 320 kg or more after grazing 3 to 5 months. Although this operation is typically a pasture program, this study considered supplementation with cottonseed meal as an alternative to offset a decrease in animal gains observed toward the end of the grazing season (Bement 1970).

Modeled Cases

Four cases were modeled with 2 selling strategies and 2 stocking densities (Table 1). In cases 1 and 2, half of the herd may be sold between 5 September and 17 October, with the remainder sold on 17 October 17 (traditional selling date). Cases 3 and 4 provide that half or all of the herd may be sold between September 5 and October 17, with the remainder of the cattle (if any) sold on October 17. The stocking density was 0.33 animals/ha (16 yearlings in a 48.6 ha pasture) for cases 1 and 3, and 0.49 animals/ha (24 yearlings in a 48.6 ha pasture) for cases 2 and 4. The selected pasture size was not typical of yearling operations, but it could easily be adjusted to the pasture size of an existing ranch. Empirical equations for vegetation and animal growth were determined by regression analysis, and assembled into the DP model. A generalized DP package (Labadie et al. 1982) was used to maximize net present value for the 4 cases previously described.

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The Dynamic Optimization Model

The model used here was dynamic in the sense that it optimized the sequence of decisions throughout the grazing season, in contrast to static models that optimize over the entire grazing season. The dynamic optimization model had 3 variables: (1) standing crop of vegetation (SCV) ranging from 400 to 620 kg/ha with 20-kg/ha intervals; (2) average animal weight (AWT) ranging from 235 to 350 kg with 5-kg intervals; and (3) number of animals per area (D) with 2 discrete values, 24 or 12 animals per 48.6 ha (high stocking density) and 16 or 8 animals per 48.6 ha (low stocking density). These discrete numbers are related to zero and 50% herd sales. Each of these state variables is related to 3 fundamental processes: forage growth, animal growth, and change in number of animals in the operation. Eleven stages were designated from late May to mid October. The beginning of stage 11 was only considered as the outcome of stage 10. A stage (subscript t) was defined as a 14–15 day period. At the beginning of that period, a decision (control u), defined below was made and applied to the rest of that period (Fig. 1). Changes in number of animals depended upon the selling alternatives selected during the grazing season; these alternatives were considered in concert with supplementation alternatives. The DP recursive relation can be written as follows:

\[ F_t(AWT_t, SCV_t, D_t) = \max \left( (P_1 \cdot V_{w_t} - \text{TOTSUPP}_t, P_2, \text{VLABOR}_t) \right) \]

where \( P_1 \) and \( P_2 \) were the steer prices in the Omaha market (USDA 1984) and cottonseed meal (USDA 1985) at stage t, respectively; \( \text{TOTSUPP} \) was the weight of cottonseed meal fed to cattle during stage t (according to Bement 1970); \( \text{VLABOR} \) was the cost of veterinary and labor expenses for stage t. This value was calculated as 30 cents animal/day without supplementation, and 35 cents with supplementation. Fixed outlay (FO) was the initial cash paid for the steers (USDA 1984), this amount was zero for all stages. This value was calculated as 30 cents animal/day without supplementation, and 35 cents with supplementation. The minimum of 2 ratios, crude protein intake (CPINT) to crude protein needs (CPN) and digestible energy intake (DEINT) to digestible energy needs (DEN) was the relative limiting nutrient (Senft et al. 1984) for that stage. The following relation:

\[ Y_{u_t} = f(AWT_t, D_t, u) \]

where \( Y_{u_t} \) depended on number of animals per pasture (D), the average weight (AWT), and the decision u.

Average animal weight at stage t (AWT_t) in equation 1 was defined by the following equation:

\[ AWT_t = AWT_{t+1} + ADG_t \cdot NODAYS_t \]

where Z was an adjusting factor for grazing conditions (discussed below), and the relative limiting nutrient (Senft et al. 1984) for a specific weight (RLN_{AWT}) was defined by the following expression:

\[ RLN_{AWT} = \min \left( \frac{CPINT}{CPN}, \frac{DEINT}{DEN} \right) \]

The minimum of 2 ratios, crude protein intake (CPINT) to crude protein needs (CPN) and digestible energy intake (DEINT) to digestible energy needs (DEN) was the relative limiting nutrient (Senft et al. 1984) for that stage. The optimum forage intake (AVEINT_t) was estimated with Conrad’s et al. (1964) equation multiplied by a linear decreasing function:

\[ AVEINT_t = \left( 0.0107 \cdot AWT_t \right) - \left( \frac{CPINT}{CPN} \cdot \frac{DEINT}{DEN} \right) \]

where DMD was in-vitro dry matter digestibility of forage. According to Bourdon (1983), Conrad’s equation (first term in parentheses) underestimated the ratio fecal dry matter to live weight (0.107) in growing steers. To compensate for underestimation, G(AWT_t) was a linear decreasing function from 1.1 to 1.0, representing 10% extra forage intake for 235 kg steers and 0% for animals weighing 340 kg or heavier. Forage crude protein and in-vitro dry matter digestibility (Table 2) plus crude protein and in-vitro dry matter digestibility (Table 2) plus crude protein and

Table 2. Percentage of crude protein (CP) and in-vitro dry matter digestibility (DMD) for the grazing season at CPER

<table>
<thead>
<tr>
<th>Period</th>
<th>Stage</th>
<th>CP</th>
<th>DMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/30</td>
<td>6/12</td>
<td>1</td>
<td>12.9</td>
</tr>
<tr>
<td>6/3</td>
<td>6/36</td>
<td>2</td>
<td>12.0</td>
</tr>
<tr>
<td>6/27</td>
<td>7/10</td>
<td>3</td>
<td>10.8</td>
</tr>
<tr>
<td>7/11</td>
<td>7/24</td>
<td>4</td>
<td>9.8</td>
</tr>
<tr>
<td>7/23</td>
<td>8/7</td>
<td>3</td>
<td>9.4</td>
</tr>
<tr>
<td>8/8</td>
<td>8/21</td>
<td>6</td>
<td>9.1</td>
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<tr>
<td>8/22</td>
<td>9/4</td>
<td>7</td>
<td>8.1</td>
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<td>9/5</td>
<td>9/19</td>
<td>8</td>
<td>7.1</td>
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<tr>
<td>9/20</td>
<td>10/3</td>
<td>9</td>
<td>6.8</td>
</tr>
<tr>
<td>10/4</td>
<td>10/17</td>
<td>10</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Interpolated figures from monthly data reported in Dean and Rice (1975), Senft (1983), Uresk and Sims (1975) and Vavra et al. (1973).
Beginning of planning horizon: May 30.

Stage = 14-15 days

End of planning horizon: October 17.

Fig. 1. Temporal framework of the planning horizon and the sequence of stages in the DP.

Results and Discussion

Partial Sales Marketing Strategy

Cases 1 and 2 allowed partial sales at a given marketing date and utilized low and high stocking density, respectively. Optimal trajectories of the variables are described below.

Standing crop of vegetation followed similar trajectories for the 2 cases. During the first 3 stages there was a net increase in the standing crop (Fig. 2) corresponding to 97 mm of precipitation at CPER during that period (52% of the total rainfall during the grazing season). The standing crop of vegetation during the next 3 stages was stable while in the last 4 stages declined. The final standing crop of vegetation was 40 (low stocking density) and 20 (high stocking density) kg/ha above the initial 400 kg/ha at the beginning of the grazing season.

Under low stocking density, the number of animals per area during the first 9 stages was 16; at the beginning of stage 10 half of the number of animals in the previous stage (D_{t-1}) minus the number of animals sold in stage t (D(Y_{t,o})).

where AVEINT_t is average forage intake (kg per day per animal). Average forage intake was limited to 80% of estimated AVEINT_t if a forage reserve level was lower than 380 kg/ha as a consequence of forage overutilization. This condition defined a step function that related a threshold value of standing crop to animal intake, this assumed condition was used in the absence of empirical data for this relationship. Therefore, AVEINT_t was dependent on the decreasing forage quality (DMD in Table 2) and possible shortage of forage (below the forage reserve level). In the absence of empirical relationships that relate stocking densities and forage quality dynamics, this model assumed that stocking density per se did not exacerbate the temporal decreasing trend in forage quality.

Density of animals at stage t (D_t) in equation 1 was defined by the following equation:

$$D_t = D_{t-1} - D(Y_{t,o}),$$

where the number of animals per pasture at stage t (D_t) was equal to the density of animals in the previous stage (D_{t-1}) minus the number of animals sold in stage t (D(Y_{t,o})).
Fig. 2. Optimal trajectories of the state variables under partial sales strategy.

the animals were sold and 8 were kept to be marketed at the beginning of stage 11. Animals per area under high stocking density were kept at 24 during the first 8 stages and decreased to 12 during stages 9 and 10. This suggested that greater grazing pressure prompted the decision to reduce the pressure one stage sooner than in the case of low stocking density. In order to avoid reduced forage intake and decreased animal growth, the stocking density was adjusted, resulting in partial sales.

Initial average animal weight was 235 kg. In both cases the average daily gain was 1.1 kg per animal per day during the first 3 stages, and later decreased to 0.71 kg per animal per day for the rest of the grazing season, resulting in a final weight of 350 kg at the beginning of stage 11. Supplementation was provided during stages 9 and 10 in both cases, preventing average daily gains of only .36 kg per animal per day.

Steer prices ($/cwt) in 1984 followed the trend of the last 4 years (USDA 1984), as shown in Figure 3. Declining livestock prices towards the end of the grazing season must be taken into consideration since they affect net revenue. Using a low stocking density, livestock were sold early not because of forage overutilization but because it was more profitable to sell at stage 10 than stage 11. This was an economic decision based on the principle that additional revenue of retaining livestock one more stage must be larger or equal to the additional costs associated to that stage. This decision rule applies for discrete cases such as the ones presented here (i.e., the changes in returns and costs are calculated between stages or periods of time, as opposed to infinitesimal changes). For example, the revenue of selling steers at the beginning of stage 9 was $467.02 (330 kg divided by .454 kg/pound multiplied by $64.25 and divided by 100 pounds); similarly, the revenues for selling cattle at stages 10 and 11 were $479.54 and $477.35, respectively. Total variable costs without supplementation at the beginning of stage 9 were $33.90, while total variable costs with supplementation the beginning of stages 10 and 11 were $39.36 and $44.44, respectively.

The difference between the change in revenues and the change in total variable costs between stages 9 and 10 was $7.06 (($479.54-467.02)-(39.36-33.90))$. The analogous difference between stages 10 and 11 was $7.27 (($477.35-479.54)-(44.44-39.36))$. At the beginning of stage 9 it was decided to retain the steers until stage 10 with $7.06 net return per stage; in contrast, at the beginning of stage 10 it was decided to sell the animals to avoid $7.27 net loss per stage. The net present value per 48.6 ha pasture at the end of stage 9 was $1,437 and $2,160 for low and high stocking densities, respectively (Table 3).

Table 3. Net present value ($) per 48.6 ha pasture at CPER under different marketing strategies and stocking densities.

<table>
<thead>
<tr>
<th>Marketing Strategy</th>
<th>Partial Sales</th>
<th>Total Sales</th>
<th>Difference Between Total and Partial Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>1437</td>
<td>1529</td>
<td>58 (3.8%)</td>
</tr>
<tr>
<td>Stocking Density</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2160</td>
<td>2293</td>
<td>87 (3.8%)</td>
</tr>
</tbody>
</table>

*These calculations did not consider rent for land, depreciation of machinery or land investments.

The values in parentheses are the percent differences between total and partial sales strategies with respect to total sale strategy.

Total Sales Marketing Strategy

Cases 3 (low stocking density) and 4 (high stocking density) allowed all of the animals to be sold at a given marketing date. As in the cases with partial sales strategy, SCV had a similar trend for both stocking densities. Under high stocking density, the SCV at the beginning of stage 10 was 400 kg/ha, equal to the initial SCV (Fig. 4). Under this marketing strategy, the forage resource was utilized close to the threshold level of 380 kg/ha. The number of animals per area was constant until all animals were sold at the end of stage 9 for the 2 stocking densities.

Initial animal weight was 235 kg. Under both stocking densities, animal growth was 1.1 kg per animal per day during the first 3 stages, decreasing to 0.71 kg per animal per day in the remaining stages. Supplementation took place in stage 9, and final weight was 340 kg at the beginning of stage 10.

Steer sales at the beginning of stage 10 were determined by the
The combination of decreasing yearling steer prices and declining forage quality towards the end of the grazing season (Fig. 3), a similar situation to that under partial sales strategy and low stocking density. The net present value per 48.6 ha pasture at the beginning of stage 10 was $1,529 and $2,293 for low and high stocking densities, respectively (Table 3).

The difference in the net present value per pasture between total and partial sales strategies was larger for high stocking densities ($87=2,293-2,160) than the difference in the net present value per pasture between total and partial sales strategies for low stocking densities ($58=1,529-1,437). These differences with respect to loss related to the retention of 5% of the herd at the end of stage 10 and partial sales strategy, early sale minimized net return losses for cases, except under high stocking density and partial sales strategy, under the partial sales strategy.

Conclusions

The combination of decreasing yearling steer prices and declining forage quality towards the end of the grazing season dictated early sales in spite of the possibility of livestock growth. Supplemental feed was necessary to maintain quality during the grazing season. It was a selected control as long as declining steer prices determined the early sale of cattle. Net present values per pasture were slightly larger for the total sales strategy than the partial sales strategy using both low and high stocking densities.