Optimizing the Calf Mix on Range Lands with Linear Programming

BRUCE M. WOODWORTH

Highlight: When faced with the decision of placing calves on two or more ranges, is there an optimum pattern? This study has shown the answer to be yes. A linear programming analysis was used to incorporate differences in the growth rates of steers and heifer calves on two ranges with different costs to find the optimum allocation of calves. The proposed allocation would have resulted in over a 4% increase in profitability over that achieved by the actual random allocation.

Ranch owners or managers are quite often afforded the opportunity to place their calf crop on two or more different ranges. Each range is generally unique with respect to two characteristics of concern to the rancher. The first of these is the cost incurred by placing the calves on a particular range. The second characteristic is the rate of weight gain realized by the animals on that range. In essence there is a gradation from extremely lush, nutritious, but expensive land to very barren, but cheap, land. In light of these differing characteristics how should the calves be allocated to each range? More specifically, is there an optimum allocation pattern?

The question was formulated as a profit maximizing problem where the general model is that profit is equal to revenue minus cost, which is to say:

\[ P = R - C \]  

The analytical technique used in the study is linear programming. Linear programming requires that the problem be expressed as a linear objective function subject to one or more linear constraints. The objective function in this case was derived from the profit model and the constraining conditions are based on the number of calves in several categories. The major difficulty in the study was to formulate the objective function. This required an identification of all relevant income and cost data. For the technique to be appropriate, of course, it is necessary to verify that the variables are linearly related. This verification was made and the variables were found to be so related.

Background

In one actual situation, an Oregon rancher was faced with allocating his herd to two ranges. One was his own land, referred to hereafter as the “inside range,” and the other was land leased from the Bureau of Land Management, referred to hereafter as the “outside range.” For this particular ranch, it was a given condition that grazing permits were available. For other ranches, this may not be the case. This, however, in no way diminishes the general applicability of the results of this study. The fundamental task is that of conducting an objective evaluation of the differences between range lands so that a given herd can be optimally allocated. It is entirely feasible, in fact, that the methodology to be presented here could be used to determine if a rancher should seek or keep grazing permits.

The data for this study represent a 5-month period of time from May 1 to October 1, 1971. The general operating procedure of the ranch can be thought of as beginning on or about October 1 of the preceding year. At this time the entire herd, which was grazing both ranges, was brought onto the inside range. Shortly thereafter, and continuing up to the beginning of the time period in question, sales of young calves took place. Also during this period, from October to May, calves were born. It is this crop of calves that is the subject of this study.

In as much as all animals lived under identical conditions, which is to say they all incurred the same costs and had equal opportunity for weight gain, the October to May period of time was eliminated from the study. In addition, herd bulls and future breeding stock were not considered influential in the allocating decision. In other words, it was assumed that the sole activity of the ranch is to produce cattle for sale at one specific time with no regard for future sales. This is not considered a restrictive assumption, and with respect to future breeding stock this can be incorporated readily into the analysis. Reference will be made to this point in a subsequent section.

The herd was allocated to the ranges by animal units with one animal unit being a cow and her calf. Since it is common practice to place all data on a monthly basis, the primary variable was one animal unit per month or one AUM. At the May 1 starting point there were 888 AUMs to be allocated to the two ranges. The actual allocation, which took place prior to this study, was 425 AUMs to the outside range and 463 AUMs to the inside range. With respect to the outside range, there were grazing permits available for 506 AUMs. With respect to the inside range, it was the opinion of the rancher that his land would support a maximum of 506 AUMs.

1 I would like to express my appreciation to Mr. Cameron Curtiss for the collection of the necessary data for this study.
Revenue Function

In order to formulate the revenue function\(^2\) for the profit model, the basic data needed was the amount of weight gained by the calves. The decision to use weight gain was based on the assumption that, although ranges may differ significantly in the quantity and quality of forage available, these differences would be reflected ultimately in the ending weight of the calf. Initially, the only comparison was a simple one of the outside range versus the inside range. Subsequent study showed that in addition to differences between ranges, there was also a difference depending upon the type of calf, whether it was a steer or a heifer. It is common knowledge that the two types of animals do have different rates of weight gain. This differential is apparently very slight, however, which often suggests an intuitive conclusion that it is an immaterial factor in the decision process. The rancher in question felt this was the case, and the allocation of the calves was essentially a random process. In anticipation that this could be an important factor, and also to be completely objective, it was decided to incorporate these differences into the analysis by creating a separate variable for each type of calf on each range.

The variables in the study are as follows:

\[
\begin{align*}
X_1 & \quad \text{the number of steer AUMs to be placed on the outside range} \\
X_2 & \quad \text{the number of heifer AUMs to be placed on the outside range} \\
X_3 & \quad \text{the number of steer AUMs to be placed on the inside range} \\
X_4 & \quad \text{the number of heifer AUMs to be placed on the inside range}
\end{align*}
\]

These variables are the unknowns of the study and the objective is to determine their values.

Since these two factors, calf type and range location, affect the ultimate sale weight, ideally the data should be the net change in gross weight for each type of calf on each type of range. On the other hand, the average age of the calves at the time of allocation is only 1.5 months. At such an early age, there is little significant weight differential between the two types. It was estimated that the average weight for both types was 150 pounds at the time of allocation.

Upon round-up and sale, the average total weights for each category were as follows: steers outside, 430 lb; heifers outside, 280 lb; steers inside, 417 lb; and heifers inside, 259 lb. Subtracting the average initial weight from the average sale weight gives the net increase per calf type as follows: steers outside, 250 lb; heifers outside, 259 lb; steers inside, 267 lb; and heifers inside, 247 lb. To put the weight gain on a monthly basis, the above net increases were divided by five, which is the length of the time period in the study. The net increase by calf type per month is: steers outside, 56 lb/month; heifers outside, 52 lb/month; steers inside, 53 lb/month; and heifers inside, 49 lb/month. It should be noted that the net increases per month are average figures. It is quite typical that calves will gain weight more rapidly at the beginning of the grazing period. As the availability of forage diminishes, the rate of weight gain does also. In this study the calves were placed on a range for the entire 5-month season, hence the use of averages is acceptable. It would be acceptable in all cases where the quantity and quality of forage remain adequate throughout the entire season.

To complete the revenue function, the net monthly weight gains are multiplied by the selling price per pound. For the actual use of proposed methodology, it would be necessary to have a forecast of the prices. In this study these were known and at the time of sale the commercial prices were $0.385/lb for steers and $0.34/lb for heifers. Note that the price differential between calf types gives further justification for distinguishing between them on each range. The revenue function for the model is

\[
R = 21.56 X_1 + 17.68 X_2 + 20.40 X_3 + 16.66 X_4
\]

This function states that for every steer AUM placed on the outside range, that is \(X_1\), total revenue will increase $21.56; for every heifer AUM placed on the outside range, \(X_2\), total revenue will increase $17.68, and so forth.

Cost Function

The cost data considered to be significant in making the allocation decision were the variable costs associated with each range. The fixed costs for the ranch were not included in the analysis because these costs were of such a nature that they would be incurred regardless of how the calves were allocated. Therefore, as long as the size of the initial herd remains constant, fixed costs will not influence the decision. If the objective of the study had been to determine the optimum herd size, fixed costs would be an important factor. Since this was not the objective of the study, eliminating these costs from consideration is justified. Some of the elements in this fixed cost category are labor, the operation of certain major pieces of equipment, taxes, and depreciation. Obviously these are the major cost elements of a ranch operation, and to ignore them gives a somewhat distorted view of the profitability of the operation. In spite of this, the fact remains that they do not influence the allocation decision.

Outside Range

There were two elements of variable cost for calves on the outside range. The most obvious and easy to determine was the price charged by the leasing agency. This figure was $2.18 per AUM.

The second cost was that incurred due to animal mortality. The number of deaths is a variable element, of course, depending on a number of factors. For this study, however, it was assumed to be proportional to the number of AUMs placed on the range. Actual losses were converted to a percentage of the total AUMs, and this figure was termed a loss rate. There is a loss rate for both cows and calves, and, of course, the cost of the loss of an animal also depends on whether it is a cow or a calf. Finally, the figure must be converted to a monthly basis.

Taking the cows as an example, there were six deaths during the period of the study out of a total 425 units on the range hence:

\[
\frac{6}{425} = 0.01412
\]

This is a percentage figure which means that 1.412% of the total number of cows on the range will perish. The percentage

\[\text{JOURNAL OF RANGE MANAGEMENT 26(3), May 1973}\]
of Equation (3) is then divided by the length of time on the range, which was 5 months:

\[
\frac{0.01412}{5} - 0.00282 \quad (4)
\]

This means that 0.282% of the cows will perish per month. Next the percentage figure of Equation (4) is multiplied by the value of a cow, which in this case was $307:

\[
0.00282 \times 307 = 0.8667 \quad (5)
\]

The figure is now the proportional cost per month due to the death of a certain number of cows. To obtain the actual cost, the figure from Equation (5) is multiplied by the, as yet unknown, number of AUMs to be placed on the outside range. In terms of the variables of the problem this becomes:

\[
0.8667(X_1 + X_2) \quad (6)
\]

Following the same procedure for the calves, where there were 18 deaths valued at $146 each, gives a death rate of:

\[
1.2366(X_1 + X_2) \quad (7)
\]

The total cost of the outside range, \(C_o\), is the sum of the rental charge and the death rate figures from Equations (6) and (7):

\[
C_o = 1.28(X_1 + X_2) + 0.8667(X_1 + X_2) + 1.2366(X_1 + X_2)
\]

\[
= 4.2833(X_1 + X_2) \quad (8)
\]

One cost not included in this cost function but which could be very important for other ranches is a transportation cost. In many cases the proposed grazing land is so far away from the ranch that the cattle must be moved by mechanical means. This would become, then, an additional variable cost. In this study, the movement of the cattle to the outside range was accomplished by a drive.

**Inside Range**

With respect to the inside range, which is the private property of the rancher, there are innumerable costs which could be considered in formulating the total cost function. As pointed out previously, however, they are largely fixed and independent of the number of cattle being run. Costs which might depend on the number of cattle would be such things as feeder, fertilizer, and certain equipment. For the ranch in question, it was estimated that this element of variable cost amounted to $1.50 per AUM.

The other major variable cost for the inside range was the loss of some animals. This cost was computed in the same manner as for the outside range with the only differences being the actual losses and the total number of AUMs involved. There was a total of 463 units on the inside range. For the cows, there was one death, hence the proportional cost per month multiplied by the, as yet unknown, number of AUMs to be placed on the inside range becomes:

\[
0.1326(X_3 + X_4) \quad (9)
\]

For the calves, there were five deaths, hence:

\[
0.3154(X_3 + X_4) \quad (10)
\]

The total cost for the inside range, \(C_i\), is the sum of the variable costs and the death rate figures from Equations (9) and (10):

\[
C_i = 1.50(X_3 + X_4) + 0.1326(X_3 + X_4) + 0.3154(X_3 + X_4)
\]

\[
= 1.948(X_3 + X_4) \quad (11)
\]

**Total Cost**

The total cost function for both ranges is obtained by combining the individual cost functions of Equations (8) and (11). The result is:

\[
C_{o+i} = 4.283X_1 + 4.283X_2 + 1.948X_3 + 1.948X_4 \quad (12)
\]

**The Linear Programming Formulation**

Having obtained all of the necessary cost and revenue data, the next step in the analysis is to convert this information into the linear programming format. The ultimate format will vary depending on the intended method of calculation, but basically it requires a mathematical expression of the study objective together with any restrictions which exist.

**Objective Function**

The first component of this format is the objective function which shows the per unit contribution being made by each variable of the problem to the overall objective of the problem. The objective in this study is profit, and the variables are the two calf types on each range. The per unit contribution to profit is determined by following the basic profit model of Equation (1) which is to subtract costs from revenue. Taking the revenue function, as determined by Equation (2) and subtracting the total cost function, as determined by Equation (12), results in:

\[
P = R - C_{o+i} = (21.56X_1 + 17.68X_2 + 20.40X_3 + 16.66X_4) - (4.283X_1 + 4.283X_2 + 1.948X_3 + 1.948X_4)
\]

\[
= 17.277X_1 + 13.397X_2 + 18.452X_3 + 14.712X_4 \quad (13)
\]

**Constraining Factors**

The second component of the linear programming format is comprised of the constraint equations. A constraint equation represents a restriction of some sort on the decision to be made. In this study, then, there are certain restrictions on how the cattle can be allocated to the two ranges. One set of constraining conditions reflects the total number of animal units which each range can support. For the outside range this is:

\[
X_1 + X_2 \leq 506 \quad (14)
\]

. . . where 506 is the number of grazing permits available. In other words the total of heifer and steer AUMs cannot exceed 506. For the inside range the constraint is:

\[
X_3 + X_4 \leq 506 \quad (15)
\]

. . . where 506 is the maximum number of animal units which this range can support. This constraint reflects the size of the range and the quality and quantity of the forage available.

A second set of constraining conditions reflects the total number of animal units in the herd. There are three restrictions in this set: the total herd, the number of steers, and the number of heifers. For the herd:

\[
X_1 + X_2 + X_3 + X_4 = 888 \quad (16)
\]

. . . where 888 is the size of the initial herd. An estimate of
the composition of the herd showed that for steers:

\[ X_1 + X_3 = 472 \]  
\[ X_2 + X_4 = 416 \]

where 472 is the initial number of steer calves in the herd. For heifers:

\[ X_3 + X_4 \leq 354 \]

This constraint represents a subjective estimate of the most efficient manner of operating the ranch.

**The Linear Programming Model**

The complete formulation of the problem is:

\[ \text{maximize: } P = 17.277 X_1 + 13.397 X_2 + 18.452 X_3 + 14.712 X_4 \]
\[ \text{subject to: } X_1 + X_2 + X_3 + X_4 = 888 \]
\[ X_1 + X_2 + X_3 + X_4 = 888 \]
\[ X_1 + X_3 = 472 \]
\[ X_2 + X_4 = 416 \]
\[ X_3 + X_4 \leq 354 \]

In this particular format the problem is ready for submission to a computer, which was the solution procedure used in this study. For hand calculation it is necessary to perform some additional manipulation on the constraints. This will not be done here, but there are a number of excellent sources which explain the procedure. Stockton (1971), for example, has a complete yet elementary treatment of general linear programming, while Hadley (1962) is more suitable for the technically inclined. Both these sources are industrially oriented, whereas Heady and Candler (1958) treat the subject more in an agricultural context. For an example of the use of linear programming in range management see Van Dyne (1966).

**Model Results**

The results of the linear programming analysis show the values for the four variables to be:

\[ X_1 = 382 \text{ (steers to the outside range)} \]
\[ X_2 = 0 \text{ (heifers to the outside range)} \]
\[ X_3 = 90 \text{ (steers to the inside range)} \]
\[ X_4 = 416 \text{ (heifers to the inside range)} \]

A comparison of these figures with the random allocation showed the two to be different. The answer to the primary question in this study, therefore, is an unqualified yes. There is an optimum pattern of allocation for the two ranges. In addition to these values, the linear programming analysis also included the profits generated by the optimum allocation. The gross margin was $14,434.30. This figure is quite misleading, of course, due to the use of masked data.

An attempt to determine whether the difference between the two allocation decisions was truly significant or merely superficial, reference was made to the true revenue, cost, and calf data for the period in question. The actual allocation generated a gross margin of $8,594.54; whereas if the allocation had been made according to the linear programming model, there would have been a gross margin of $9,015.00. This amounts to an increase in the gross margin of over 4% per month.\(^4\)

Obviously the utilization of linear programming for determining the heifer-steer mix on a range is an effective means of improving the profitability of a ranch operation. It is true that this study was concerned with only one ranch, but the potential benefit exists for all ranches faced with the same, or a similar, decision. There will be variation, of course, among ranches and ranges, which will necessitate a period of data collection. As this study has shown, the primary data are weight gain, cost, and revenue for each animal unit.

Another major advantage of linear programming is the ease with which the technique can be applied. As formulated in this study, the solution can be obtained by hand calculation in less than an hour. If computer services are available, the solution can be obtained in approximately two seconds at a cost of $1.66 for computer time.

**Insuring Specific Allocations**

At this point it is appropriate to recall the statement made earlier concerning future breeding stock. Looking only at the coefficients of the variables \(X_2\) and \(X_4\) in Equation (13), which refer to the heifer calves, there is an implication that the majority of heifers will be placed on the inside range due to the larger contribution to profit. At this stage of the development there is no guarantee that this will happen, but the possibility does exist. On the other hand, the weight gain data show that heifers will gain more rapidly on the outside range. It is important to have future breeding stock gain rapidly in order to reach breeding size as quickly as possible. A conflict exists, then, between the short range objective of profit maximization and the long range objective of herd maintenance. It may very well be that the importance of rapid weight gain could override the lower profitability which would result from placing heifers on the outside range.

It is by means of a constraint equation that a rancher can insure achievement of his long range objective. For example, if it was decided that 50 heifers were to be retained for future breeding stock, and that these animals must go to the outside range, then the appropriate constraint would be:

\[ X_2 \geq 50 \]

Incorporating this equation into the linear programming formulation, although not done in this study, would insure that at least 50 heifers would be placed on the outside range. In a similar manner the rancher can place his cattle on any location he wishes for special purposes.

**Literature Cited**


---

\(^4\) Only gross profits can be determined due to the exclusion of fixed costs.

---

*The percentage increase should be considered as a minimum value because, again, of the exclusion of fixed costs.*

---

178 JOURNAL OF RANGE MANAGEMENT 26(3), May 1973