A Proposed Method of Determining Cattle Numbers in Range Experiments

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In range grazing studies having a prescribed degree of utilization as part of a treatment, it is necessary to estimate the number of animals to be placed on each experimental pasture. Several experiments at the Rocky Mountain Forest and Range Experiment Station have shown consistent relationships between the number of animals grazed, the amount of herbage produced, and the utilization at the end of the grazing period. This relationship can be used as a guide for stocking experimental pastures.

A general formula that has shown promise at several locations for developing a mathematical relationship is shown as formula 1 (Figure 1). In formula 1, U is the observed percent utilization of the principal forage species or class at the end of the grazing period, P is the production in pounds per acre of the principal forage species or class, and S is the stocking during the period expressed as acres per animal unit. The a, b_1, and b_2 are constants derived in computation of the multiple regression.

This formula, or variations of it, has given high multiple correlation coefficients in several grazing studies in which the method was tested.

At Cebolla Mesa, in northern New Mexico, seven years of data from small seeded pastures of crested wheatgrass gave multiple correlation coefficients of 0.97, 0.98 or 0.99 for each of three study pastures, and for all pastures combined.

Cebolla Mesa is an example of the simplest use of the formula. The areas grazed were all small, contained only one principal forage species, and distribution problems were at a minimum. If the relationships existed, good ones would be expected, and were found.

In Colorado, on high-altitude summer range on Black Mesa, four years of data were collected on six experimental pastures that ranged from 110 to 394 acres. Multiple correlation coefficients of 0.97 or higher were obtained for all pastures. Here the formula was applied to the production and utilization of Idaho fescue alone, the principal forage species, although many species made up the forage. Stocking was based on the area of open grassland within the pasture. The aspen, which received light use, and the dense spruce forests did not enter into the computation.

Black Mesa was a somewhat more complicated situation than Cebolla Mesa for several reasons: the pastures were larger; a variety of species made up the forage, but only one was used in the production and utilization estimates; distribution problems were present; and not all the area in each pasture was used by the cattle. The grazing was closely controlled, however, and production and utilization were intensively sampled. Good relationships were obtained.

Data from the Jornada Experimental Range near Las Cruces, New Mexico, were used to study the possibility of such relationships being developed on larger ranges where sampling of production and utilization was on a more extensive basis. Fifteen years of data from six ranges (1,780 to 83,960 acres) gave multiple correlation coefficients exceeding 0.94, when utilization and production data for black grama alone were used in formula 1 and the acreage per animal-unit month was limited to the black grama type. Similarly good correlations were obtained when production and utilization of all grasses were combined, and the tobosa type as well as black grama type were included.

On the Santa Rita Experimental Range in Arizona, a much more complicated utilization was studied. Seven years of data were used from 16 experimental pastures that varied in area from 650 to 5,500 acres. Vegetation varied from grassland to shrub types. Grazing patterns within pastures were diverse and annual grasses, perennial grasses, forbs and shrubs provided the forage. Grazing time and methods varied by pasture. In these studies, a related formula (for-

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$$U = a + b_1 \frac{1}{p} + b_2 \frac{1}{S} \tag{1}$$

$$\frac{1}{U} = a + b_1 \frac{AG}{S} + b_2 \frac{PG}{S} \tag{2}$$

$$S = \frac{b_1 AG + b_2 PG}{\frac{1}{U} - a} \tag{3}$$

For a general method of deriving confidence limits for S, let equation 2 be written in the standard form:

$$\hat{Y} = \overline{Y} + b_1 (X_1 - \overline{X}_1) + b_2 (X_2 - \overline{X}_2) \tag{4}$$

where $Y = \frac{1}{U}$ $X_1 = \frac{AG}{S}$ $X_2 = \frac{PG}{S}$

Then the variance of the regression estimate, $\hat{Y}$, is given by

$$\text{Var}(\hat{Y}) = s^2 \left[\frac{1}{n} + c_{11} (X_1 - \overline{X}_1)^2 + 2c_{12} (X_1 - \overline{X}_1)(X_2 - \overline{X}_2) + c_{22} (X_2 - \overline{X}_2)^2 \right]$$

where s^2 = residual error of regression 4 ; n = number of observations;

c_{11}, c_{12}, c_{22} = elements of the inverse matrix.

Let Y be regarded as true value of $\hat{Y}$ for given X_1 and X_2.

Then $t = \dfrac{\hat{Y} - Y}{\sqrt{\text{Var}(\hat{Y})}}$ which has a "t" distribution with n-3 degrees of freedom. (5)

Substituting for $\hat{Y}$ and Var ($\hat{Y}$) in equation 5, and squaring leads to

$$\left[\overline{Y} + b_1 (X_1 - \overline{X}_1) + b_2 (X_2 - \overline{X}_2) - Y \right]^2 = t^2 s^2 \left[\frac{1}{n} + c_{11} (X_1 - \overline{X}_1)^2 + 2c_{12} (X_1 - \overline{X}_1)(X_2 - \overline{X}_2) + c_{22} (X_2 - \overline{X}_2)^2 \right] \tag{6}$$

Expanding and collecting terms gives

$$b_1^2 (X_1 - \overline{X}_1)^2 (1 - g_{11}) + 2b_1 b_2 (X_1 - \overline{X}_1)(X_2 - \overline{X}_2)(1 - g_{12}) + b_2^2 (X_2 - \overline{X}_2)^2 (1 - g_{22})$$

$$+ 2(\overline{Y} - Y) \left[b_1 (X - \overline{X}_1) + b_2 (X_2 - \overline{X}_2) \right] + (\overline{Y} - Y)^2 - \frac{t^2 s^2}{n} = 0 \tag{7}$$

where $g_{11} = \dfrac{t^2 s^2}{b_1^2} c_{11}$ $g_{12} = \dfrac{t^2 s^2}{b_1 b_2} c_{12}$ $g_{22} = \dfrac{t^2 s^2}{b_2^2} c_{22}$

Substituting values of AG, PG, and Y in equations 6 or 7 leads to a quadratic equation in $\frac{1}{S^2}$ or S^2; the solution gives the upper and lower confidence limits for S.

FIGURE 1. Formulae used to estimate stocking rates and the general method of deriving confidence limits of stocking estimates, an independent variable.

mula 2, Figure 1) was used. In formula 2, U is the percent utilization, S is stocking in animal units, AG is pounds of annual grass produced per acre, and PG is pound of perennial grass produced per acre.

This functional form of the equation appeared to be satisfactory for estimating stocking rates from estimates of herbage production on the range. Both low as well as high correlation relationships were found, which gave an opportunity to determine causes of poor relationships.

Multiple correlation coefficients exceeded 0.9 for half the pastures. The best correlations, with one exception, were obtained for pastures in which the grasses were the dominant forage plants and distribution of cattle was not changed from year to year. The poorest relationships were found where the grasses used in the computations

Year[1]	Basic data				In terms of equation 2		
	Utilization	Production		Stocking			
		Annual grass	Perennial grass		Y	X_1	X_2
	Percent	Pounds per acre		Animal units			
1954-55	62	83.7	272.7	20.1	0.0161	4.16	13.57
1955-56	50	437.1	273.5	19.6	.0200	22.30	13.95
1956-57	71	53.4	143.0	17.3	.0141	3.09	8.26
1958-59	22	117.2	627.5	8.8	.0454	13.32	71.31
1959-60	25	340.6	888.4	18.5	.0400	18.41	48.02
1960-61	63	10.1	233.2	19.4	.0159	.52	12.02
Mean	--	--	--	--	.0252	10.30	27.86

[1] Data for 1957-58 not included because of unusual spring growth that prevented an estimate of U.

The normal equations are given by:

$$404.2246\, b_1 + 511.7684\, b_2 = 0.345491$$
$$511.7684\, b_1 + 3327.0857\, b_2 = 1.743303$$

which leads to the solutions: $b_1 = 0.239 \times 10^{-3}$; $b_2 = 0.487 \times 10^{-3}$;

$$c_{11} = 3.072 \times 10^{-3}\ ; \quad c_{12} = -0.473 \times 10^{-3}\ ; \quad c_{22} = 0.373 \times 10^{-3}$$

Analysis of variance:

Source of variation	DF	MS	F
Due to regression	2	0.000466	93.20 (significant at 95 percent level)
Residual	3	.000005	
Total	5		

R = 0.992

From equation 4, the prediction equation for utilization is given by

$$Y = 0.0252 + 0.000239(X_1 - 10.30) + 0.000487(X_2 - 27.86) = 0.000239X_1 + 0.000487X_2 + 0.0091$$

Assume production of AG = 150 pounds per acre; PG = 500 pounds per acre. To obtain the estimated stocking rate for 40 percent utilization, substitution in equation 3 gives

$$S = \frac{[\,0.239\,(150) + 0.487\,(500)\,]\ 10^{-3}}{\frac{1}{40} - 0.0091} = 18 \text{ animal units.}$$

Equation 4 should be recomputed each year with additional data for that year and solved as above for stocking, S.

To obtain confidence limits, when $t_{.05} = 3.183$ at three degrees of freedom, substitute in equation 6 to obtain

$$g_{11} = 2.722992 \qquad g_{12} = -0.205751 \qquad g_{22} = 0.079627$$

$$1 - g_{11} = -1.722992 \qquad 1 - g_{12} = 1.205751 \qquad 1 - g_{22} = 0.920373$$

$$(0.239)^2 \left(\frac{150}{S} - 10.30\right)^2 (-1.722992)\,10^{-6} + 2\,(0.239)\,(0.487)\left(\frac{150}{S} - 10.30\right)\left(\frac{500}{S} - 27.86\right)(1.205751)\,10^{-6}$$

$$+ (0.487)^2 \left(\frac{500}{S} - 27.86\right)^2 (0.920373)\,10^{-6} + 2\left(0.0252 - \frac{1}{40}\right)\left[\,0.239\left(\frac{150}{S} - 10.30\right) + 0.487\left(\frac{500}{S} - 27.86\right)\right] 10^{-3}$$

$$+ \left(0.0252 - \frac{1}{40}\right)^2 - \frac{(3.183)\ (0.5)\ (10^{-5})}{6} = 0$$

Combine similar terms in $\frac{1}{S^2}$ and $\frac{1}{S}$ and then cross multiply by S^2 to obtain

$$0.000224S^2 - 0.008273S + 0.073343 = 0$$

The roots $S_1 = 15$, $S_2 = 22$, are the 95-percent confidence limits for the estimate of 18 animal units.

FIGURE 2. Example of computations of stocking estimates and confidence limits.

were not the major forage or where the cattle management within the pasture was changed every year. For example, the poorest relationship ($R = 0.32$) was for a pasture used to test the influence of placing salt and salt-meal mixture at and away from water on a four-year rotation. Variations in placement and kind of mineral and feed supplements changed the pattern of use on the pasture each year. Intermediate relationships were found for pastures in the driest parts of the range where shrubs and winter annual forbs made up a considerable proportion of the forage. The shrubs and forbs were not measured when yearly forage production or utilization was determined. These observations lead to the conclusion that, to use the relationship, major forage species must be measured and a consistent management program must be followed within a pasture.

In practice, data from past years are used to compute an average relationship among variables for each pasture or range area. A separate relationship is needed for each area because each varies as to cattle distribution patterns, relative amounts of forage types, and relation of the forage species measured for production and utilization to all the forage produced and used within the pasture. The regression equation can properly be used to estimate the percent utilization that will result from a given yield and rate of stocking in the pasture because utilization (U) is the dependent variable subject to random error.

An investigator more often desires to know the number of animals to place on an experimental pasture to arrive at desired degree of utilization. This requires solving the regression equation for S (stocking), which is an independent variable subject to selection, and setting confidence limits for this estimate. In particular from equation 2 we have equation 3 (Figure 1).

Substitution of given values of AG, PG, and U results in the estimate of S. A general method for deriving confidence limits for S may be obtained as shown in Figure 1. Figure 2 is an example of the computation of stocking, S, and confidence limits for a pasture on the Santa Rita Experimental Range.

To date the procedure has been used only to estimate grazing capacity of experimental pastures where rather intensive data on herbage production and utilization of the main species have been available. Its best use is where production estimates can be made before livestock are placed on the range, and utilization estimates can be made after the livestock are removed and before growth begins. The method can be used, however, to estimate stocking to meet a given herbage production. On Black Mesa, for example, stocking rates are adjusted at three-year intervals, the estimate each three years being made of the number of animals required based on a year of average herbage production. The most extensive test of the method was with the Jornada Experimental Range data in which good production and utilization records were available for large pastures. Herbage production and utilization were determined each year on four 50-foot transects per section (640 acres). The good relationships obtained on the Jornada suggest possible use on range allotments as an approach to estimating grazing capacity. Trials of this kind are recommended.