The Use of Regression in Range Research¹

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There are two kinds of tools which range technicians can carry with them when they do research -mechanical and statistical. Some mechanical tools are simple. The simple ones may be crude, such as an ax handle for measuring stubble heights, or more refined, such as a meter stick. The complicated tools, like the capacitance meter, are designed for the task of taking measurements accurately and objectively, yet cheaply and easily. Statistical tools have a parallel classification. Some of the simple ones are crude, such as the use of the average to describe a population, and some are more refined--affixing the fiducial limits to the average. The complicated statistical methods are powerful tools for the analysis of data after the measurements have been taken. They increase the precision of research and facilitate the interpretation of experimental results and survey data.

Regression analyses are statistical methods which fit into the category just described. They are useful in their simplest form-linear regression-but even more useful with the complexity of multiple regression. Certain uses of regression are made-to-order for many problems in the field of range management. Studies of forage utilization, grazing management, reseeding, fertilization, brush control, chemical analyses, digestion trials, range economics-each of these brings to mind a large number of perfect set-ups for regression analyses.

Regression is perhaps better understood when illustrated than when defined. Most people are acquainted with the concept of re-1. Paper presented at the Eighth Annual Meeting of the American Society of Bange Management, January 25-28, 1955, San Jose, California. gression even though they do not realize it is that. On the penny scale outside of the drug store is a table reading something like this:

MEN					
HEIGHT	WEIGHT				
5'2"	130 lbs.				
5'3″	134				
5'4"	139				
5'5"	144				
5'6"	148				
5'7"	153				
5'8"	158				
5'9"	164				
5'10"	168				
5'11"	172				
6'0''	177				
6'1"	182				
6'2''	187				

This table shows the *regression* of weight on height for the human adult male population. It is useful to express this graphically in the form of a regression line, which is the heavy diagonal line in Fig. There is an *independent vari*-1. able: height. This is arrayed along the horizontal axis. The dependent variable, weight, is perpendicular to this. The idea of independence and dependence usually, but not necessarily, denotes cause and effect. The height which a person has attained, which in turn is dependent on other factors such as age, heredity, or childhood diet, determines to a large extent what he weighs.

The independent variables are fixed. They can be selected, that is, they do not have to be taken at random. Now, in our example, there is a distribution of weights for any selected height. The distribution has a mean that is actually the average weight for all men of this height.

The data in Fig. 1 make up a *scatter diagram*. Plotted are the heights and weights of 17 range technicians residing in the Berk-

eley, California area. The finer diagonal line is the regression line computed from the 17 positions on This line represents the graph. the average weight-per-height of these individuals better than any other straight line we could draw. The line was determined by the method of least squares, which simnly means that if we sum the squares of each of the 17 distances from the points up to or down to the line, that sum is smaller than the sum for any other line.

With this knowledge of the characteristics of independent and dependent variables and the meaning of regression, little imagination is needed to visualize relationships between rate of seeding and density of grass, rate of nitrogen application and yield of forage, age of ruminant and digestibility, and so on for any biological attribute that is affected by measurable environmental phenomena.

As used in the height-weight example, regression is simple, but it can be extended to six or more uses. These are: (1) predicting an unknown value of a dependent variable by interpolation or extrapolation of data; (2) estimating a difficultly obtained value of a char-



FIGURE 1. Regression of weight on height for the American adult male population (heavy line) and for 17 Berkeley, California range technicians (light line).

acteristic more precisely by measuring an auxiliary variate which can be found quickly and cheaply; (3) reducing the error variance of designed experiments and increasing the relative information through statistical control; (4) determining the degree of independence of a factor; (5) testing the form of a relationship, whether linear or exponential (curvilinear); and (6) determining the proportion of the increase or decrease of a dependent variable which is contributed by each of several measurable independent factors.

The object is not to go into the statistical aspects of regression, but merely to point out, with appropriate examples, the ways in which regression can be applied to range research.

Predicting Unknown Values

The man at the carnival who can guess your weight within two pounds has had much experience at that game. He sizes up his sucker, height being an important factor in formulating his prediction, and allows a four pound leeway (confidence limit). The greater his length of experience, the narrower he can make his confidence limits and still operate his concession at a good profit.

A regression equation is a mass of past experience expressed in algebraic form. It can be used to predict what will happen under certain specified conditions. The accuracy of the prediction is entirely dependent upon the breadth of the past experience. Thus, the larger the sample used in the compilation of the equation, the narrower the confidence limits may he Confidence limits are an expression of the number of chances out of a hundred that your prediction will be right.

Using the regression lines in Fig. 1, if you know of a man who is 6 feet tall, you can guess his weight to be 178 pounds plus or minus a few pounds. The wider this leeway, the greater proportion of time you will be right. Now if you learn that he is a Berkeley



FIGURE 2. Regression of number of brush seedlings per square foot surviving the first summer of growth on foliar density of ryegrass (from Schultz, Launchbaugh, and Biswell, 1955).

range technician he is likely to be five pounds heavier, but you can't say this with too much confidence since there were only 17 men in this sample compared to the thousands involved in the other regression line. Moreover, it is safer to predict weights on range men between the heights of LR and ALH (interpolation) than beyond those extremes (extrapolation), for reasons given later.

Another way to use the regression line is to determine what the normal or expected weight should be, then compare this with the actual. If the deviation is great, it may be well to look for reasons. Note that comparison with the sample mean weight does not yield such insight. For example, HFH is only four pounds over the group or population average weight of 171 pounds but for his height he is 28 pounds over while RAM is 21 pounds under the population average and 35 pounds under the average for his height. If the data represented 17 pigs, the farmer who owned them might use this information and select the HFH, JIM, and ALH types for breeding stock and deworm those way off the line in the lower right corner.

The divergence of two regression lines is sometimes of interest. The reasons why, among these research men, the ectomorphs are more ectomorphic and the "heavyweights" are heavier than generally found in the adult American male population may be supplied with a few more independent variables or they may have Freudian explanations. At least, in this case, height does not explain everything. Most likely, the two lines in Fig. 1 are not significantly different.

Bona fide range problems find the most common and practical use of regression to be in its predictive value. Ordinarily you predict the unknown dependent variable when the size of the independent variable is stipulated. A graph using some familiar data on the grass density-brush seedling relationship (Schultz, Launchbaugh, and Biswell 1955) will illustrate how this can be done in reverse (Fig. 2). Each increment of density of ryegrass is associated with an increase in mortality of the brush seedlings. On the average, at 38 percent density, mortality is 100 percent. Suppose we would like to maintain about one seedling per 10 square feet for browse, what should we strive for in grass density? The answer can be interpolated from the graph.

Regression Estimates

Sometimes we are interested in a measurement which is extremely



FIGURE 3. Percent use as determined by the weight of herbage removed in relation to percent of ungrazed plants of all important grasses on the Santa Rita Experimental Range, Tucson, Arizona. The circles are observations of ungrazed plants grouped by 5 percent intervals (from Roach, 1950).

costly to obtain with any degree of Protein analyses, for accuracy. example, take a lot of time and the equipment necessary to run them is not always available. Measures of range utilization are difficult to get in terms in which a range technician or a rancher may have confidence. Use of the linear regression estimate increases the precision and decreases the cost. By measuring an auxiliary variate which can be found quickly and cheaply, we can estimate more precisely the value of a characteristic obtained with great difficulty. We do not have any interest in this auxiliary variate itself; we are interested only as it is related to the dependent variate in question.

Let us illustrate this principle with two examples. It would be well to have a quick method of estimating crude protein in range forage. Ranchers could use it to decide when supplemental feeding should be started and the amount of supplement needed. A close relationship has been found between the crude protein in forage on longleaf-pine bluestem ranges in Louisiana and the free moisture in that forage (Campbell and Cassady 1954). Free moisture is relatively easy to determine. This relationship can be expressed in the form of a regression line. When used in conjunction with the known protein requirements of beef cattle, the moisture content of the forage will indicate the amount of protein that must be supplemented.

The other example is one of estimating perennial grass utilization on semidesert ranges by percentage of ungrazed plants (Roach 1950). After establishing the relationship between the percent of plants grazed and the percent utilization as a whole (Fig. 3), the latter value can be estimated merely by counting the number of grazed and ungrazed plants on representative samples. Roach claims that this simple method, when compared with the height-weight method, cuts field time by half and office computation time by at least three fourths; and it gives utilization estimates within 5 percent of those gotten the other way. The percentage of ungrazed plants itself is of no interest except that it helps to estimate utilization.

Statistical Control

Plant physiologists who are bequeathed with unlimited funds have elaborate laboratories and greenhouses where nearly every essential feature of the environment can be controlled. Thus, an experiment can be reduced to only one variable such as growth. With complete control over all factors, there should, theoretically, be no unexplained error encountered in the experimentation. Range ecologists have two strikes against themthey never are bequeathed with unlimited funds and if they were, they would fall short in controlling most factors of the outdoor environment, as the rainmakers can attest. So their research is redolent with what is called experimental error.

While he cannot control the factors, the range ecologist can often measure them and these measurements enable him to use regression for increasing the information available in his data. This is statistical control. In fact, the ecologist may be better off than the physiologist because in many cases statististical control is more desirable than experimental control. First, the actual situation is studied, not one produced artificially; secondly, a far greater range of observations can be made which broadens the foundation for inference; and finally, one learns how two quantities instead of one vary, singly and together-the factorial approach.

All this can be illustrated with some data on brush seedlings that are competing with certain species of annual grasses (Table 1). The table is excerpted from one containing many other species of grasses and legumes.

The number of brush seedlings per 100 square feet remaining alive after growing one season with these annual grasses varied with the grass species—significantly. But the number of brush seedlings that came up on these plots was not the same. Stated anthropomorphically, it is harder for cereal rye to reduce 275 seedlings to 0 than for ryegrass to reduce 125 since 125 is already much closer to 0. (Biologically, this premise may be incorrect because intraspecific competion may be greater among the 275.) Because of the uncontrollable discrepance in numbers, the original seedlings were counted and this auxiliary variate was used to adjust the fall numbers. Column 3, after the adjustment, shows the number of seedlings which would occur in fall if the same number had started on all the plots in the spring.

Some grasses grow better than others on different sites so grass species occur at varving densities. Would these species be equally good competitors if they grew at equal densities? Again, column 3 was adjusted so that column 4 shows the numbers of seedlings that would occur if the original seedling numbers were the same and the grass densities the same. Now, two variables have been controlled. We have divested the species of extraneous factors and perhaps we can compare them strictly on the basis of some physiological character such as transpiration ratio.

The method whereby these adjustments are made is called *anal*ysis of covariance. Other examples and explanations of the method are in the range literature (Pechanec 1941; Blaisdell 1953).

Test for Independence

One cannot always be sure that one factor is dependent on another even though logic says they are, and conversely, that two factors are independent when no cause and effect are indicated. It is a matter of degree. There is a statistical test which gives you reason to say, at a given level of

Table 1. Number of bru	sh seedlings establish	ed in competition	with annua	l grasses
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	Number of Brush Seedlings per 100 Sq. Ft.				
Competing Species					
	(1)	(2)	(3)	(4)	
	Spring	Fall	After 1st	After 2nd	
	Count	Count	$\mathbf{Adjustment}$	Adjustmen	
Cereal Rye	275	10	1	1	
Red Brome	233	14	9	13	
Annual Ryegrass	125	4	7	17	
Rose Clover	152	14	16	27	
Soft Chess	163	37	36	33	
Check	177	47	45	32	

significance, that one variable is independent of another, or that one is dependent on the other. We can say that in the height and weight example, weight of range technicians is dependent on their height, and be right in saying that unless a 1 out of 25 chance has come about. The statement was made earlier that a relationship of dependency may exist but it may not necessar ily mean that a cause and effec relationship exists. Any relation ship can be a tool for predictior or analysis purposes but the de cision as to whether it is causal or not comes only after detailed re



FIGURE 4. Comparison of costs of controlled burns and of wild fire suppression i northern California, 1947-1948 (from Sampson and Burcham, 1954).

search of the biological or logical kind rather than from mathematical manipulation of data.

An example of the question of dependence can be illustrated with data taken from an article on costs of controlled brush burning (Sampson and Burcham 1954). Fig. 4 is reproduced from this article. The second curve from the top implies that if a certain size of area is controlled burned, say, for example, 280 acres, it will cost about \$1.20 per acre. In short, the cost per acre is directly dependent on size of burn. The top curve implies that this is also true of wildfires. However, it is much more reasonable to assume that the size of the wildfire is dependent on the amount of money devoted to its suppression. Thus, while the curve drawn for "cost of wildfire suppression" is not invalidated for predictive purposes, there may be some theoretical objections to comparing it with the curve for "total cost of controlled burns."

Test for Linearity of Regression

Not always do two variables go off in the same direction as do height and weight. Especially in biological phenomena, sigmoid, bell-shaped, or cyclic relationships are found. With growth phenomena a factor may increase the rate of growth until that factor has reached an optimum magnitude, and then growth decelerates. This relationship is also described by a regression line but a curved instead of a straight one. A curvilinear regression equation is difficult to compute, but from the practical as well as the academic viewpoint it is important to know whether a relationship is curvilinear or actually linear.

This can be illustrated with a hypothetical fertilizer experiment (Fig. 5). Six rates of nitrogen were applied to replicated plots, 0 to 500 pounds per acre in 100 pound increments. Forage production for each plot has been plotted on the graph. The broken line connects the averages for the treatments. Linear regression tells us



FIGURE 5. Regression of forage yield on rates of nitrogen fertilizer application for a hypothetical experiment. Dots are individual plot yields; circles connected by broken line are treatment means; solid straight line, linear regression line; double line, curvilinear or quadratic regression line; dotted lines, projections or predictions beyond the highest treatment level (outrapolation)

ment level (extrapolation).

A statistical test indicates that the regression in this case is quadratic (one-humped). One can readily understand that if more nitrogen is added without adding additional amounts of phosphorus, potassium, or water, not only will the extra nitrogen do no good but it will actually even have a depressing effect on the yield.

This test for linearity, then, is a useful tool for many aspects of range research. Straight line relationships are the exception rather than the rule where growth phenomena are involved.

Multiple Regression

Why are two plants not exactly alike even when they are of the same species and grow only 12 inches apart? A host of factors contribute to their difference. Among these are the number of hours head start one had as a seedling, the difference in food reserves in the seed, amount of time one is in the shade of the other, and so on. Conceivably 480 or more measurements of that nature would explain quite fully the variation between the two plants. Let us extend this idea to a current problem in game range management.

It would be easy to determine the production of available deer browse if we knew that all brush plants produced the same amount of forage: simply count the plants and multiply by the weight produced per plant. Chamise plants in the brushlands of northern California grow on varied sites and their size and vigor vary so much that an extremely large and costly sample must be taken to yield a reliable answer. Nor is it a simple matter to clip the available forage on a single bush. It might be better to choose a number of other characteristics of the plant, preferably some that can be measured easily and with objectivity, find out how much each contributes to the total variation in browse production between plants. The few characteristics which contribute the most can then be used in range investigations to determine browse production.

To illustrate, the following "independent" variables were measured on each of 32 chamise plants: average height (X_1) , number of live stems (X_2) , foliage crown diameter (X_3) , root crown diameter (X_4) , length of average leader (X_5) , and length of longest leader (X_6) . The dependent varible was available annual growth (Y). Many hours of calculations resulted in this equation:

 $\begin{array}{c} \mathbf{Y} \\ (\text{predicted}) = -366.60 - 23.35 \mathbf{X}_1 \\ + \ 7.60 \mathbf{X}_2 \ + \ 1.86 \mathbf{X}_3 \ - \ 37.96 \mathbf{X}_4 \\ + \ 5.15 \mathbf{X}_5 \ + \ 69.77 \mathbf{X}_6. \end{array}$

This equation is so impressive that it cannot be presented in graphical form since our comprehension is limited to three dimensions. The variables X_2 , X_3 , X_5 , and X_6 are associated with vigor; any increase in either of these will increase Y. The variable X_1 is negative; any height over 4 feet is superfluous for deer, since they browse only to that height. Indeed, shorter plants provide a top surface in addition to the sides where browse is available. The large and negative X_4 is a function of age. When all other factors are held constant, the plant with the larger root crown is the older, more decadent, and less productive. This fact was not readily discernible from routine methods of analyzing the data, e. g., correlation.

The six measurements account for only 38 percent of the variability. Most of the other 62 percent lies in the remaining 473 characteristics which could be measured and analyzed simultaneously with standard IBM machines.

It is not difficult to see how both fundamental and practical range problems can be clarified by the approach of multiple regression. The multitude of factors contributing to the processes which we call competition, soil formation, or plant succession, and the many forces and conditions that influence animal production, plant vigor, or the economics of range management are still too much a matter of subjective debate. If these factors are real, they are measurable; and, if measurable, they are interpretable. Multiple regression affords a good way to interpret them.

To conclude with the same whimsical example with which we began, multiple regression would permit us to predict with great accuracy the weights of Berkeley range technicians. For, besides height, their weights are dependent on age, amount of field work or desk work they do, size of their expense accounts, number of milk shakes they imbibe, and so on, until all the variation in their weights is accounted for.

Six ways in which regression can be used in range research have been discussed. There are others that may well be applicable in this field although not widely used. Numerous textbooks explain how to do the calculations but that has not been in the scope of this paper.

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Some Effects of Date of Planting, Depth of Planting and Fertilization on the Performance of Five Important Native Grasses of Texas

Abstract of thesis submitted in partial fulfillment of the requirements for the degree of Master of Science, Department of Range & Forestry, Agricultural & Mechanical College of Texas, 1955.

Five important range grasses of Texas were planted during the fall and winter of 1953-54, on an abandoned cultivated field near College Station, Texas. Plantings of big bluestem, silver bluestem, Indian grass, side-oats grama, and little bluestem were made on four dates: November 21, January 2, February 6 and March 5. Plantings were made of each species on the surface and at depths of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $\frac{1}{2}$ inches. On May 27, three fertilizer treatments were made: top dressing, band dressing applied at the rate of 90-45-0 per acre and no fertilizer. All plots were weeded at various intervals.

Plantings of big bluestem resulted in relatively poor stands with few plants. A majority of these plants resulted from the February and March plantings. Establishment of plants was greatest for the $\frac{1}{4}$ - and $\frac{1}{2}$ -inch depth plantings.

Relatively large numbers of silver bluestem plants were established at all planting dates, with maximum in March and minimum in February. Planting at depths of $\frac{1}{4}$ and $\frac{1}{2}$ inch resulted in greater establishment than at other depths. Planting at $\frac{1}{2}$ -inch depth resulted in failure for all dates of planting.

Indian grass produced a comparatively substantial number of plants for all dates of planting. This grass showed the least variation in plant numbers for the various dates of planting of all the grasses observed. Plantings at depths of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ inch resulted

in from fair to excellent stands for all dates of planting. All 1¹/₂-inch depth plantings rated poor. Results from surface plantings were erratic, and ranged from no plants to good stands.

Side-oats grama produced the greatest number of plants of any of the grasses. The November and January plantings resulted in poor emergence whereas the February and March plantings resulted in good to excellent emergence. Planting depths from $\frac{1}{4}$ to 1 inch for the February and March plantings resulted in stands that rated excellent. Surface plantings rated fair to excellent. Planting depths of $\frac{11}{2}$ inch resulted in poor stands for all dates of planting.

Counts of little bluestem were considerably higher for the February and March plantings than for the November and January plantings. Planting depths of $\frac{1}{4}$ and $\frac{1}{2}$ inch for this grass resulted in the best stands of any of the depths. Surface and $\frac{3}{4}$ -inch depths resulted in poor to fair stands, 1-inch depths in poor to no stands, and $\frac{1}{2}$ -inch depths in no stands. Fertilizer applied as a top dressing to the five

Fertilizer applied as a top dressing to the five grasses appeared to give better results, as related to height of plant and herbage production per foot of vegetation, than the band dressing and no fertilizer. Big bluestem was the only grass not reflecting these results. Indications were that band dressing was slightly less favorable than no fertilizer. This may have been due to poor soil moisture conditions at the time of application and for a considerable period thereafter.—JAMES E. ANDERSON, Department of Animal Husbandry, New Mexico Agricultural & Mechanical College, State College, New Mexico.