

Economic Criteria for Determining Optimum Use of Summer Range by Sheep and Cattle¹

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THE primary purpose of this article is to outline an economic framework of analysis wherewith the most profitable combination of sheep and cattle on a given range can be determined (Hopkin, 1954). A recent issue of this Journal carried a most interesting article by C. Wayne Cook, (Cook, 1954), wherein he presented his estimates of grazing capacity of a given mountain range when grazed by both sheep and cattle with various numbers of each, and when each class was grazed alone. His calculations were based on estimates of the vegetational composition and estimates of the

utilization by cattle and sheep. From these estimates the forage acre factor was computed for each class of livestock and the combined grazing capacity of the given range for several different combinations of sheep and cattle was estimated.

According to the basic assumptions of the forage acre factor and grazing capacity any one of the combinations of sheep and cattle suggested by Cook is equally desirable from the standpoint of forage production or "proper" carrying capacity. He does not specify which combination is preferred, but merely shows that a greater number of animal units can be grazed when both cattle and sheep are combined on the range than when each class

is grazed separately. A secondary purpose of the article is to submit the very interesting data outlined by Cook to the analysis of the economic model in order to determine the adequacy of his suggested enterprise relationships.

The Economic Framework

Agricultural economists have been working for some time on the economic problem of selecting a most profitable crop or combination of crops from among the possible crop sequences. Out of this experience has come the logic and method of determining the optimum enterprise combination for a given set of resources (Heady, 1952, pp. 201-275). The economic principles of optimum enterprise combination apply as well to range resources as to any other kind of resources.

Consider the problem of determining the optimum combination of cattle and sheep on a given range site. Although there are some areas that are better adapted to one class of livestock than another, for the

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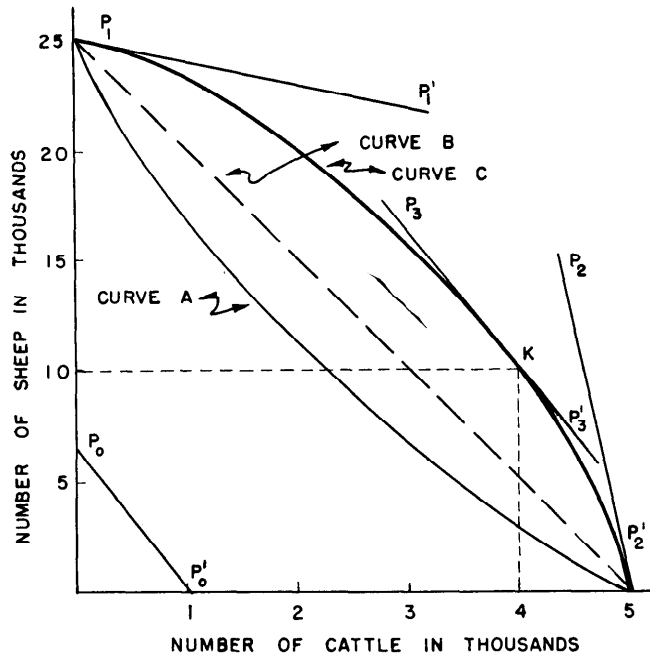


FIGURE 1. Hypothetical enterprise relationship between sheep and cattle.

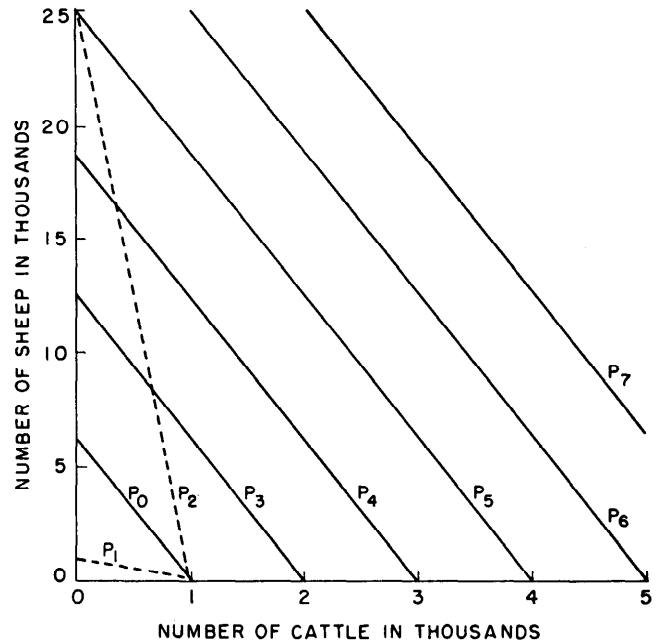


FIGURE 2. Iso-revenue lines for various price relationships of sheep and cattle.

most part, both classes can utilize the same range. The economic problem of determining the optimum combination to maximize long-time profits turns on two basic relationships that need to be known (or estimated with sufficient confidence to warrant a decision), viz., the physical relationship or marginal rate of transformation, and the price relationships reflecting the relative preferences of consumers.

The physical relationships

The first of these basic relationships (the marginal rate of transformation or product substitution), merely outlines the number of physical units of one class of livestock (sheep) that would have to be removed from the range in order to permit one unit of the other class (cattle) to be added to the range *and still leave the range in as good condition as it previously was*.

This physical relationship is illustrated graphically for a hypothetical situation in Figure 1, where three different types of physical relationships are shown together for comparison. Curve B (the straight line)

shows a constant marginal rate of substitution. It is plotted by first showing the maximum number of cattle (with no sheep) that could be grazed on the range, based on sound range management. This is assumed to be 5,000 head, and is plotted as (5000, 0) on the grid coordinate. Next, determine the number of sheep (with no cattle) that can be grazed on this same range (again, without injury to the plant species). In Figure 1 this is assumed to be 25,000 head and is plotted (0, 25,000). If the rate of substitution between cattle and sheep on this range is constant we can represent such a relationship by drawing a straight line between the two plotted points. This produces Curve B, which is the locus of points showing the maximum number of sheep that can be grazed on this hypothetical range when any specified number (from 0 to 5,000) of cattle are being grazed. According to Curve B, the condition of the forage would be the same whether 1,000 head of cattle and 20,000 head of sheep, 3,000 cattle and 10,000 sheep, or 5,000 cattle and zero sheep were being grazed.

This is the kind of relationship that is implied in most range management research and recommendations when it is considered that one cow is equivalent to five sheep for all combinations on a given range. Cook (1954) very neatly points out the error of the assumption of constant marginal rates of product substitution.

Curve A shows a decreasing marginal rate of substitution. The curve is convex to the origin indicating that no combination of sheep and cattle would be as productive as would one class by itself. Where this relationship exists, the most profitable solution would always be where only one class of livestock was grazed. This type of relationship was implied in the early days of the range when the belief predominated that sheep and cattle could not use the same range. This kind of thinking still dominates the decisions of many ranchers, grazing associations and public land administrators, since more grazing land is allocated singly to either sheep or cattle than is grazed jointly by both.

A much more reasonable assump-

tion is that an increasing marginal rate of transformation exists, for reasons that were emphasized by Cook (1954, p. 10).

Many grasses on summer ranges are used only sparingly by sheep, especially late in the grazing season after they become stemmy, whereas, cattle eat most grasses rather readily during most of the grazing season. In addition cattle consume many forbs and shrubs with avidity but generally more complete use of these plants is made by sheep. Therefore, more effective use of summer range might be made by sheep and cattle grazing in combination.

This relationship is represented by Curve C of Figure 1, which we will want to examine in detail. (For the time being we will ignore the straight lines P_1P_1' , P_2P_2' , and P_3P_3' , and focus on the physical relationship implied in Curve C). Again, we assume that when devoted entirely to cattle, a maximum of 5,000 head of cattle can be grazed year after year without damage to the range. Curve C shows that by removing just a few head of cattle, a substantial number of sheep can be added (the slope of the curve is very steep at the lower end) and still leave the range in the same condition as when grazed by 5,000 head of cattle. This occurs, of course, because the sheep are consuming the plant species that were under-utilized by the cattle. As more cattle are removed, however, relatively fewer sheep can be added, per unit of cattle removed, without injury to the range. (The traditional 1:5 ratio is reached when Curve C is parallel to Curve B). At the other extreme, the maximum number of sheep that can be grazed when zero cattle are grazed is 25,000 head. At this point several head of cattle could be added by removing just a few head of sheep (the slope of the curve is very flat), but as sufficient cattle are added to eat the coarse, dry feed, they come more into competition with the sheep. Note that the supplementary relationships (flat and

steep parts of the curve) occur at the extremes, and that the slope of the curve becomes more constant toward the center. This will be important later on. The relationship of Curve C is logical based on (1) traditional range management principles (as indicated above by Cook) and, (2) the law of diminishing returns (variable proportions).

The curves of Figure 1 are, of course, hypothetical. They are the assumed physical relationships that might exist between two enterprises (cattle and sheep) in the use of a given quantity of resources (a hypothetical range with a fixed quantity of labor and capital), and are called iso-resource curves. Only an increasing marginal rate of substitution, where the curve is concave to the origin (Curve C) is logically consistent for ranges where both classes of livestock can graze.

It must be emphasized that the curves are production possibility lines. They describe only physical phenomenon and have their origin in the physical science of range management. They can be derived in two ways. First, they may be obtained from physical experiments where sufficient combinations of sheep and cattle are observed over time under controlled experiments designed for curvilinear regression analysis. Obviously, results from experiments of this nature are forthcoming only after several years. In the meantime, the relationships may be reasonably approximated by quantitative and qualitative judgments of competent range management technicians based on the preference index figures for each kind of livestock and on the range inventory and range condition. The same procedure used in estimating grazing capacity for one type of livestock (cattle) could be used in estimating grazing capacity for various combinations of sheep and cattle. The assumption that the livestock will graze over the entire range area without concentrating in local areas

may be more nearly true for combinations of sheep and cattle than for one livestock species grazing alone. The actual shape of the curve will depend on the relative proportion of forage that is preferred by each class of livestock. The general shape of Curve C (Fig. 1) or Curve F (Fig. 3) will hold for all range units where both cattle and sheep can graze.

Irrespective of how the curve is estimated, it outlines the possible combinations that leave the range in the same condition, but it provides no criterion for selecting the preferred combination from among the possible combinations. Before a selection can be made we must have choice criteria that we can apply to our model.

Price Relationships

The choice criteria we will use in this case are the price relationships of the two commodities (sheep and cattle). These will be determined by means of iso-revenue lines. For example, if the market price of a steer (of the weight and quality we are assuming in the above hypothetical situation) is \$187.50, then from selling 1,000 head we could derive \$187,500. The number of head of sheep we would have to sell to obtain the same revenue depends on the price of the sheep. If the market price of sheep is \$30 per head it would take 6,250 head to bring the same revenue as obtained from 1,000 head of cattle. When these two points are connected with a straight line (see line P_0 of Figure 2), this line becomes an iso-revenue line, with every point on it representing a combination of sheep and cattle which, when sold at the assumed market prices, will provide \$187,500. The same reasoning can be applied to 2,000 head of cattle and it will be found that (at the assumed prices) 2,000 head of cattle or 12,500 head of sheep will each sell for \$370,000. Line P_3 , connecting these two points, represents all combina-

tions of sheep and cattle that together will return \$370,000. It is obvious that as long as the relative prices of sheep and cattle do not change all iso-revenue lines will be parallel, regardless of the amount of revenue each represents. It is equally obvious that one iso-revenue line that is further from the origin than another represents more revenue than does the latter. Thus P_7 represents more revenue than P_6 . There can, of course, be a vast number of iso-revenue lines drawn, each representing a separate quantity of revenue. The slope of the iso-revenue line gives us the price ratio of the two commodities—the ratio at which cattle exchange for sheep in the market. In the case of P_0 (and lines parallel to it) the ratio is 1:6.25. Line P_1 (Fig. 2) reflects a very high price for sheep. Here the ratio is 1:1. Line P_2 reflects a very low price for sheep (price ratio = 1:25).

It should be realized that the iso-revenue line refers to gross revenue and not net revenue when market prices are used. If there are substantial differences in the costs of producing and marketing the two products these differences could be considered so as to determine a "net" price ratio or an iso-net-revenue line. For purposes here we will use the iso-revenue line for simplicity. The logic would be the same in each case.

The Theoretical Optimum

We are now in position to determine the theoretical optimum combination for this range. The iso-resource curve shows the combinations of sheep and cattle that are possible without injury to the range. The iso-revenue lines show the combinations of sheep and cattle that are of equal revenue. As we move further from the origin each iso-revenue line represents a higher revenue. The optimum combination along the production possibility (iso-resource) curve to where we are on

the highest possible iso-revenue line. The solution will be, of course, when the iso-revenue line is tangent to the iso-resource curve.

Let us take an example. Assume the prices to be \$187.50 per head for cattle and \$30 per head for sheep. This is shown by line P_3P_3' of Figure 1 (which is drawn parallel to P_0P_0' so that it is tangent to Curve C). Now Curve C tells us how many sheep we can add by removing a specified number of cattle from the range (and vice versa). The slope of the iso-revenue line (the price ratio) tells us how many sheep it takes to be equal in value to cattle that are given up. In this case it takes 6.25 sheep to be equal in market value to one cattle unit. At the point where these two relationships are equal (the point of tangency—point K) profits will be a maximum. This is the optimum combination and is uniquely determined.

Under the relationships of Curve C, grazing only sheep on that range would be optimum only if the price ratio was at least as favorable for sheep as that shown by iso-revenue line P_1P_1' , Figure 1, (one sheep equal in market value to one cattle unit—see line P_1 , Fig. 2). Grazing only cattle would be optimum only if the price ratio was at least as favorable for cattle as shown by P_2P_2' , Figure 1, (25 sheep equal in value to one cattle unit—see line P_2 , Fig. 2).

Application of the Model

We will now use the above model in analyzing the data presented by Cook in order to: (1) check the suggested physical relationships for logical consistency, and (2) determine hypothetically optimum solutions for different assumed price ratios. The data were derived from grazing experiments where sheep and cattle were grazed separately on adjacent and comparable areas. The vegetational composition was determined for both ranges and the percent utilization of plant species was

estimated separately for sheep and cattle. The forage factor for sheep and for cattle was thus estimated for each species. The aggregate forage factor for sheep and cattle was computed to be .2034 and .3728 respectively. Thus, with cattle alone more animal units (based on the customary 5 to 1 ratio) could be grazed than with sheep alone. "However, if the higher forage factor for either cattle or sheep is used for each plant species the total becomes .4339" (Cook, 1954, p. 11). Cook takes this figure to represent "the forage factor for common use" and estimates a total grazing capacity of 652 animal units for that combination where 422 animal units of cattle and 230 animal units of sheep are being grazed. (At that point the ratio of sheep to cattle is the same as the ratio of their respective forage factors, or 1.83.) The calculated grazing capacities for different combinations of sheep and cattle are listed in Table 1. The first two columns are taken directly from Table 2 of Cook's report. The marginal rate of substitution of cattle for sheep is merely the ratio of the decrease in sheep divided by the increase in cattle. It simply states that, according to these data, one animal unit of cattle can be added for each .177 animal units of sheep that are removed, and the range will still be in the same condition as when grazed only by sheep. Cook's data indicates that this

Table 1. Combinations of sheep and cattle (in animal units) on the same range and marginal rate of substitution of cattle for sheep. (Data based on Cook (1954))

Cattle	Sheep	Marginal Rate of Substitution of Cattle for Sheep
0	306	—
141	281	.177
281	255	.177
422	230	.177
468	153	1.674
514	77	1.674
560	0	1.674

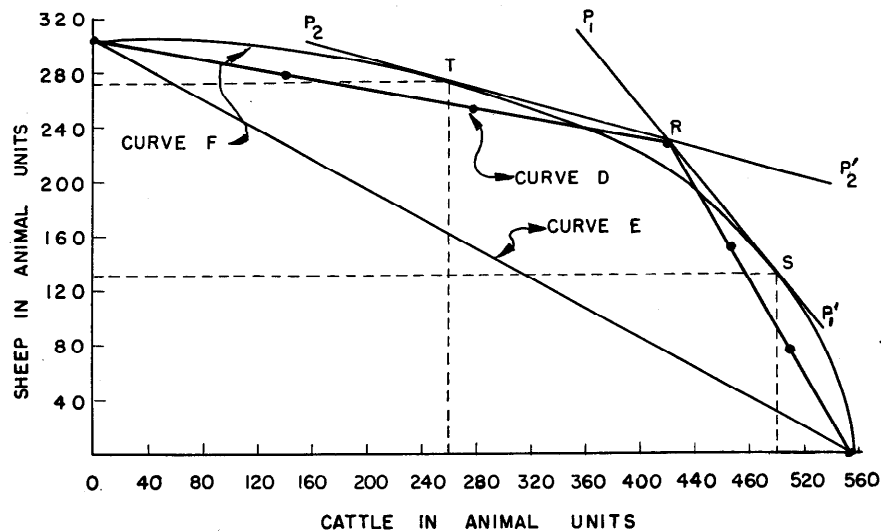


FIGURE 3. Combinations of sheep and cattle on 2,800 acres of range land in the Wasatch Mountains and optimum use for selected price relationships of sheep and cattle.

marginal rate of substitution is constant at .177 up to the point at which 422 animal units of cattle and 230 animal units of sheep are grazed. Beyond that point, however, an animal unit of cattle could be added only by reducing the number of sheep by 1.674 animal units.

Curve D of Figure 3 is obtained by plotting Cook's data (number of sheep and cattle in Table 1) on a coordinate system and connecting the seven points. Based on his interpretation of the forage factor Cook rightfully discredits the argument that sheep and cattle displace each other on the range at a constant rate. (Such a relationship would be shown by Curve E where the marginal rate of substitution is constant at $306/560 = .546$). The hypothesis of Curve D is a distinct improvement over Curve E for it does show the marginal rates of substitution to be increasing, rather than constant. However, there is nothing in the logic of range management or economics that supports the hypothesis that the marginal rate of substitution of cattle for sheep remains constant at a low rate (.177) up to a certain point (point R, Fig. 3) and then suddenly increases to 1.674, remaining constant at the new level beyond that

point. We must conclude that the shape of Curve D results from the fact that only one point (point R) was independently determined and the intermediate points were determined by linear interpolation.

Based on the reasons for common use mentioned by Cook, it is logical that when all, or most, of the livestock were cattle, a few sheep might do fairly well on those species that cattle do not utilize. At the other extreme, a few cattle might do well on those species for which sheep show little preference. As the numbers of the two classes of livestock become such that they compete directly for the important forage species, the marginal rate of substitution would not change so rapidly. Curve F is suggested as a more realistic picture of the substitutional relationship in question. (In the absence of detailed empirical observations Curve F might be only a rough approximation; it has been drawn free hand for illustrative purposes only.)

Although at first glance one may get the impression that the difference between Curve D and Curve F is so little as to be insignificant, a more careful analysis reveals otherwise. First consider the optimum combination under the assumed

market situation as indicated by the iso-revenue line P_1P_1' , Figure 3. (Here the price of cattle is high relative to the price of sheep, 1:1.2.) For Curve D the point of tangency with the iso-revenue line is at point R, indicating the optimum combination to be 422 animal units of cattle and 230 animal units of sheep. For Curve F the point of tangency is at point S (500 animal units of cattle and 132 animal units of sheep).

Next, consider the optimum combination under the market situation as indicated by the iso-revenue line P_2P_2' (cattle prices are low relative to sheep prices, 1:.275). For Curve D, the optimum solution has not moved from point R, while for Curve F the optimum solution now calls for only 250 animal units of cattle and 272 animal units of sheep (point T).

Some limitations of the model

The above model is limited, of course, by any substantial inaccuracies of the physical information that go into determining the functional relationship. This limitation is no greater for this model, however, than for any decision that is made relative to the grazing of any livestock on that range. The model assumes equal resource inputs not only of range lands but of labor and capital. This may not be true in some instances. Range sheep require closer supervision than do range cattle. Either they are under the direct watch of a shepherd or they are placed within sheep-tight fences. There may be important economies of scale associated with the production of sheep or cattle. These factors can be considered in the analysis (although they have not been in the above simple model). They are no more an inherent weakness of this method than of any alternative analytical system. Should the differences in cost structure between the two enterprises be too complex to permit their consider-

ation through the use of net revenue lines, then a more complex model would have to be used. The logic of the simple model would still direct our analysis by specifying the information needed for the analysis and by determining the statistical procedures to use in collecting and analyzing the data and in testing the hypotheses.

The problem of determining the effective price ratios for sheep and cattle has not been discussed because of space limitation. In this illustration only naive hypothetical prices have been used. It is obvious that more is involved here than a comparison of the market price of feeder steers and feeder lambs for a given season.

Some policy implications

To encourage adjustment to an optimum combination of domestic grazing, more information needs to be known about the use adaptability of the range. This, of course, is a very complex phenomenon which to discover and predict presents methodological and technical problems that challenge the plant scientist. In the meantime, decisions continue to be made that assume the relationship between two livestock enterprises grazing the same range to be linear—an assumption that seldom can be valid. However, the first need is to estimate the enterprise relationship for a given range area and then to pass this information on to the ranchers using that area. Before the adjustment could be made on public grazing land, the procedures of adjustment would have to be worked out by the administering agencies. Although

the public land agencies now tend to compute the conversion ratio separately for each allotment (thus getting away from the conventional 1:5 ratio of cattle for sheep) they still consider the conversion ratio to be constant for all combinations (Curve B, Fig. 1, or Curve E, Fig. 3).

Consider the procedure for adjustment on a given public range where only cattle have been permitted previously. Assume that a detailed range study reveals the enterprise relationship between sheep and cattle to be as shown by Curve C (Fig. 1). The assumed price ratio between sheep and cattle is shown by the slope of the iso-revenue line P_sP_s' , and the optimum combination (the goal for which both rancher and public land administrator should be striving) is represented by point K. It is thus estimated that by removing 1,000 head of cattle from the range 10,000 head of sheep can be added and still leave the range in the same condition as when previously grazed only by cattle. It then should be possible for the ranchers using this range to exchange permits by obtaining permit for ten sheep for each cattle unit given up—up to the point where the 1,000 head of cattle have been removed and 10,000 head of sheep have been added. If adjustments were to be permitted only on the 5:1 ratio normally used by the public agencies, it is unlikely that any adjustment would occur, since the price ratio is 6.25:1.

Summary

The optimum combination of sheep and cattle on a given range is

obtained by equating two independent functions: (1) the physical enterprise relationship (the iso-resource curve) which shows the combinations of sheep and cattle that can be grazed on a given range without injury to the plant species, and (2) the price relationships (the iso-revenue line). From the standpoint of "proper" range stocking, every alternative along the iso-resource curve is equally acceptable. If the costs of producing a "unit of product" are not substantially different for the two enterprises, the iso-revenue line can be determined from market prices; otherwise additional considerations must be given.

The suggested analytical model was applied to some very interesting data presented by Cook in a noteworthy contribution to the science of range management (Cook, 1954). The discussion has been directed toward a refinement of the method in order that it might be amenable to economic analysis and thus useful in making decisions pertaining to the combination of sheep and cattle on a given range site. It is an example of the need for a blending of the efforts of the physical scientist and the economist in finding better solutions to range management problems.

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