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Economically Optimal Stocking Rates: A Bioeconomic Grazing Model

John P. Ritten,[1] Christopher T. Bastian,[1] and W. Marshall Frasier[2]

Authors are [1]Assistant Professors, Department of Agricultural and Applied Economics, University of Wyoming, Laramie, WY 82071, USA; and [2]Professor, Department of Agricultural and Resource Economics, Colorado State University, Fort Collins, CO 80523, USA.

Abstract

A dynamic bioeconomic model that examines economically optimal stocking rate decisions while taking into account changes in forage availability is presented. The model presented here focuses on economically optimal stocking decisions while taking into account changes in the forage resource. The model is parameterized for a stocker operation in central Wyoming. Regardless of the scenario analyzed, the general rule of 50% utilization is determined to be an economically sound management strategy. The factors most heavily influencing economically optimal stocking rate decisions are forage growth rates and the Michaelis Constant. Both grain prices and cattle prices impact financial returns yet do not directly impact optimal stocking decisions by cattle producers.

Resumen

Se presenta un modelo bioeconómico dinámico que examina decisiones de carga animal económicamente óptimas tomando en cuenta al mismo tiempo cambios en la disponibilidad forrajera. El modelo que aquí se presenta se enfoca en decisiones de carga animal económicamente óptimas tomando en cuenta al mismo tiempo cambios en la disponibilidad forrajera. El modelo se configuró para un establecimiento de engorde en el centro del estado de Wyoming. Independientemente del escenario analizado, se determinó que la regla general del 50% de uso es una estrategia de manejo económicamente sólida. Los factores que ejercen mayor influencia en la decisión de carga animal económicamente óptima son las tasas de crecimiento del forraje y la Constante de Michaelis. Los precios de granos y del ganado ejercen un impacto en el retorno financiero, pero no influyen de modo directo las decisiones de carga animal óptima de los productores ganaderos.

Key Words: dynamic optimization, optimal stocking rates, range economics, stocker cattle

INTRODUCTION

Given the importance of rangeland resources in the provision of both forage for livestock grazing and ecosystem services, determining appropriate stocking rates has both economic and environmental consequences. Research focusing on either biologically or economically optimal stocking rates may not adequately address interactions between financial and environmental consequences of stocking decisions (Jones and Sandland 1974; Wilson and MacLeod 1991). Izac et al. (1990) compared biologic and economic optimal stocking rates and concluded that often the biological optimum stocking decisions will not coincide with an economically stable system. They have stated that ''(t)he relevance of management recommendations made to graziers and land administrators would be increased if these recommendations were based on an analysis of both the economic and the ecological stability of grazing systems over the medium to long term'' (p. 265–266). The objective of this paper is to examine a bioeconomic model linking economically optimal stocking levels to rangeland health. Special attention is given to long-term outcomes.

One of the problems with many economic modeling efforts is that, while useful to examine potential changes in the status quo, economic models often have finite time horizons, account for range resources as static and fixed, and try to determine the ''optimal'' decisions given this fixed constraint. For example, Bastian et al. (2009), Coppock et al. (2009), and Torell et al. (2001) utilize equations to account for both animal and forage states as a way to model the supply of and demand for available Animal Unit Months (AUMs) in a ranch setting over a limited time horizon (12 yr, 11 yr, and 40 yr, respectively). While these approaches illustrate the potential economic implications of impacted forage production and/or exclusion from public grazing by adding constraints or limits to available forage, the models utilized do not adequately account for the dynamic interaction between stocking decision and evolution of the forage resource.

Often, modeling efforts related to grazing decisions result in different biologically and economically ''optimal'' outcomes. This can, in part, be explained by differences in the length of planning horizon used. Manley et al. (1997) have shown that individual producers, when acting to maximize profit, generally should stock at rates that are at or below moderate levels for their study area when prices are average or below average. However, they indicated that favorable prices can lead to selection of higher short-term stocking rates that could reduce the condition of the range if maintained over a long time horizon. Hart et al. (1988) have also shown that producers

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At the time of the research, the corresponding author was a graduate student, Dept of Agricultural and Resource Economics, Colorado State University, Fort Collins, CO 80523, USA.

Correspondence: John P. Ritten, Dept of Agricultural and Applied Economics, University of Wyoming, Laramie, WY 82071, USA. Email: jritten@uwyo.edu

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looking to increase short-term profits should stock at rates up to 66% higher than rates recommended by the Soil Conservation Service. However, these authors warned that these stocking rates were likely to deteriorate the range condition over time, and producers need to compensate for this long-term trade-off when determining stocking rates in the short-term.

Models that address the interaction between present stocking densities and changes in the forage resource over time offer potential improvement over the previously mentioned economic models. While not addressing economically optimal stocking rates, Noy-Meir (1975) used an optimal control approach to model grazing as a predator-prey relationship, thereby focusing on biological stability of grazing systems. Noy-Meir (1975) forced constant herd sizes in order to determine the stability of steady states in a grazing system. By using simple state equations (or equations of resource change over time) to evaluate the potential stability of grazing systems, he was able to identify numerous general outcomes, including undergrazed steady states, overgrazing leading to either extinction or low biomass steady states, and the possibility of numerous potential steady states based on various combinations of plant growth functions and stocking density.

When considering the long-term trade-offs associated with the grazing decision, implications of stocking remain unclear. Pope and McBryde (1984) discuss how differences in land managers' optimizing behavior will affect range management. They stated that often ecologists find economic recommendations irrational, while economists may find ecological recommendations unrealistic. They went on to demonstrate that unless the planning horizon is sufficiently long and the discount rate utilized is sufficiently low, individual producers often stock public rangeland at a rate that may in fact lead to range deterioration. However, Wilson et al. (1984) stated that "(b)iologically, the optimum stocking rate varies with the rainfall, the store of forage from the previous year, the nature of the animal production system and safe utilization levels" (p. 133). These authors commented on how the economically optimal stocking rate is lower than the stocking rate that would maximize biological production per land area.

Torell et al. (1989) compared the results of a static and dynamic model of rangeland use for eastern Colorado with a 150-d grazing season. They concluded that economically optimum stocking rates over a 40-yr horizon averaged around 48% utilization of total forage over the grazing season, near the recommended 50% forage utilization rate they cite for their study area. They stated that producers who wish to maximize profits should stock at a heavier rate in order to take advantage of favorable cattle prices (while the opposite is true in years with unfavorable prices), and that in their model, the forage utilization rate varied due to varying cattle prices. They agreed with Workman (1986) in that the long-run profit motive of ranchers will not result in stocking rates that will significantly deteriorate the range.

Torell et al. (1991) further investigated interactions of stocking rates and range condition using a dynamic model calibrated for eastern Colorado. They found that ranchers do not have an economic incentive to continuously overgraze rangelands. They ignored random weather by treating each year as an average in terms of weather impacts on forage production. They stated that current animal performance drives the economic decisions and impacts on future productivity are not as important as current implications of stocking rates. They did note, however, that optimal stocking rates are decreased slightly when accounting for future forage productivity. They based this conclusion on a fixed (40-yr) planning horizon with a fixed terminal value. Their results might have differed if they had utilized an infinite planning horizon.

Standiford and Howitt (1993) utilized a dynamic optimal control model to examine the economically optimal management of rangelands in California with multiple production possibilities (grazing, wood production, and hunting revenues). While they modeled forage dynamics, they relied on oak canopy and rainfall as the primary predictors of forage production, and did not rely on standing forage as an indicator of ecological health.

While Noy-Meir (1975) demonstrated results of given actions and potential outcomes on range condition in a dynamic framework, he did not incorporate a range manager's decision behavior. Noy-Meir (1975) offered insights into the biological implications of stocking rates in a dynamic context, but also stated the importance of understanding the associated economic outcomes. Izac et al. (1990) stated the need for grazing management systems to be able to identify a "bioeconomic" optimum that enables maximization of profits while ensuring the ecological stability of a grazing system over a long planning horizon.

The previous evidence suggests that if producers fully incorporate the long-term benefits and costs of grazing decisions, they will desire a stable state that ensures both long-term rangeland health and the associated value (favorable economic outcomes) of the grazing of such a system. Therefore, the objective of this article is to analyze the long-term trade-offs of grazing management decisions. We give specific attention to management decisions that utilize the knowledge that current grazing decisions will impact future forage production. We will determine if producers' decisions should be influenced by the store of standing forage. We will also analyze how producers' decisions in such a dynamic system can be impacted by economic variables such as increased or decreased prices of cattle and corn.

METHODS

In order to account for the dynamic nature of rangeland production, a bioeconomic dynamic programming model that builds on the physical relationships presented by Noy-Meir (1975) is employed. The model encompasses not only animal performance over time and the resulting stream of discounted returns but it also incorporates how stocking decisions affect the evolving condition of rangelands. For a more detailed description of the biological portion of our optimal control model, please see Noy-Meir (1975). As our model incorporates expectations of future forage production into current period stocking decisions, it is expected that stocking rates will be lower in current periods to reduce impacts on future productivity. Producers are expected to maximize the value of land, an objective consistent with one who holds title to the land or is assured continued transferable grazing rights to the land.

Noy-Meir (1975) modeled forage growth as

$$G(V) = \gamma V\left(1 - \frac{V}{V_{\mathrm{m}}}\right) \quad [1]$$

and animal consumption is modeled as

$$C = c(V)H = c_{\mathrm{m}}\left\{\frac{[V - V_{\mathrm{r}}]}{[(V - V_{\mathrm{r}}) + V_{\mathrm{k}}]}\right\}H \quad [2]$$

The forage base is then modeled to evolve over time:

$$\frac{\delta V}{\delta t} = G - C = G(V) - c(V)H \quad [3]$$

where γ is maximum growth rate per unit of time, V is vegetation density per unit of land, and V_{m} is the maximum plant biomass for a unit of land (carrying capacity). C is total consumption per unit of land, c_{m} is the level of daily consumption associated with satiation, c is consumption per animal per unit of land, H is stocking density per unit of land, V_{r} is any ungrazeable residual or mandatory carryover biomass, and V_{k} is the plant biomass at which consumption equals half of satiation, also known as the "Michaelis Constant."

The Michaelis Constant in a grazing setting can be interpreted several ways. Cooper and Huffaker (1997) explain it as inversely related to the efficiency of a grazing animal, with a lower number translating into an animal that is able to achieve desired performance with less forage. Allden and Whittaker (1970) show that the consumption relationship to herbage allowance can be shifted due to the density of pastures. Although not measured in the Allden and Whittaker (1970) study, the same relationship is assumed to hold true for pastures of differing forage quality. As the forage resource in this model is assumed to be homogenous, a system with a lower Michaelis Constant could be analogous to a system that has higher quality forage. In other words, two identical animals will perform differently on pastures with different associated Michaelis Constants. Whichever interpretation is taken, both of these can potentially be under a producer's control, either through altering herd genetics to get more efficient grazers or by improving the quality of the pasture.

Our model is parameterized to represent a stocker operation in central Wyoming where producers determine their stocking rate and purchase animals in early summer and sell all animals in the fall. While many grazing systems are characterized more realistically as cow/calf systems, we are interested in modeling the dynamic integration of range ecology and economic implications to find the bio-economically optimal stocking rate. The inclusion of greater capital investment, the longer production period, and multiple products (i.e., steer calves, heifer calves, and cull cows) associated with cow-calf production further complicate these relationships. The production system chosen to be modeled is a stocker operation to address the trade-offs between stocking decisions and rangeland health over time. A production system that employs a cow/calf enterprise may well alter the empirical outcomes of this paper. However, this model provides a foundation on which such other systems could be modeled.

Regardless of the operational type, producers are generally concerned with profit maximization when they make their stocking rate decision. The single season return to land equation is a function of forage, stocking rate, and prices:

$$\pi(V,S,P_{\mathrm{i}},P_{\mathrm{e}},C) = \{[P_{\mathrm{e}} \cdot W_{\mathrm{e}} \cdot (1 - deathloss)] - [P_{\mathrm{i}} \cdot W_{\mathrm{i}}] - CC\} \cdot H \quad [4]$$

where P_{e} is the ending price per kilogram of the animal, W_{e} is the ending weight of an animal, P_{i} is the initial weight per kilogram of an animal, W_{i} is the weight of an animal when purchased, and CC is the seasonal carrying costs per animal. Ending weights are initial weights of animal purchased plus any gains due to grazing.

Huffaker and Wilen (1991) utilized a forage conversion coefficient to convert animal consumption to animal gain of 0.096. Our research utilizes this coefficient, which is also in line with previous studies of stocker cattle in Wyoming (Manley et al. 1997; Derner et al. 2008). Thus, total gain per animal is described as ($0.096 \cdot$ Consumption $\cdot$ Days on Pasture) over the grazing season.

A shortcoming of some previous dynamic models related to grazing is that constant price per weight is assumed over differing weight classes when evaluating optimal decisions. Prices per unit of weight, however, are not constant. Producers are faced with declining prices per unit as weight per animal increases. Cooper and Huffaker (1997) acknowledged this price slide effect, and they modeled a system where animals were purchased at 272 kg at $1.74 $\cdot$ kg^{-1}, and sold at the end of the season for only $1.43 $\cdot$ kg^{-1}. In order to account for the price slide effect in the current model, an equation forecasting prices was generated from data available for the Torrington, Wyoming, auction. This allowed for a continuous slide over the relevant range of potential weight gain. The data were received from the Livestock Marketing Information Center (LMIC; Jim Robb, LMIC, Lakewood, Colorado, personal communication, June 2007). Weekly prices were available from 1992 through 2006. It was hypothesized that grain prices would affect the price slide as well. As corn prices increase, demand for feeder cattle typically softens, and feeder cattle prices are often depressed. Thus, corn prices for the period were also obtained from LMIC. Cattle and grain prices were converted to 2008 dollars. Ordinary Least Squares regression was used to estimate cattle price as a function of weight and corn price. The estimated equation is as follows:

$$P(W_{\mathrm{end}},P_{\mathrm{corn}}) = \beta_0 + \beta_1 \cdot W_{\mathrm{end}} + \beta_2 \cdot W_{\mathrm{end}}^2 + \beta_3 \cdot W_{\mathrm{end}}^3 + \beta_4 \cdot P_{\mathrm{corn}} + \beta_5 \cdot W_{\mathrm{end}} \cdot P_{\mathrm{corn}} \quad [5]$$

where W_{end} is ending weight of cattle, P_{corn} is the price of corn, and the betas are the coefficients to be estimated. The regression returned an R^2 of 0.47 ($F = 20.36$; $P < 0.001$). Estimated coefficients are reported in Table 1. As expected, the output shows a declining price per unit of weight as animal weight is increased. This decline is less drastic when corn prices are high. When the corn price is relatively low, feedlots prefer to purchase lightweight animals and add weight themselves. However, as corn prices rise, the cost of gain for feedlots also rises, so they are less likely to pay a premium for lighter animals. Regardless of corn prices, marginal value per unit

Table 1. Estimated coefficients of the price slide equation.

	Coefficients	Standard error	*t* stat	*P* value
Intercept	12.99	8.03	1.62	0.11
Cattle weight (kg) [W_{end}]	−0.07	0.08	−0.93	0.35
Cattle weight^2	2.05E-04	2.47E-04	0.83	0.41
Cattle weight^3	−2.08E-07	2.59E-07	−0.80	0.42
Corn price ($\$ \cdot kg^{-1}$) [$P_{corn}$]	−19.35	6.74	−2.87	0.00
Corn price · cattle weight	0.06	0.04	1.44	0.15

received by cattle producers is maximized between 295-kg and 340-kg animals.

As stated above, the objective function for the producer will focus on a producer who owns title or perpetual lease rights to the grazing land. Therefore, the optimization problem is to maximize all future discounted returns to land while accounting for the effect current grazing decisions will have on future forage productivity.

$$\underset{H}{Max}\int_{0}^{T}\beta^{t}\langle\{[P_{e}\cdot W_{e}\cdot(1-deathloss)]-[P_{i}\cdot W_{i}]-CC\}\cdot H\rangle dt \qquad [6]$$

$$s.t.\frac{\delta V}{\delta t}=G-C=G(V)-c(V)H \qquad [7]$$

Estimated parameters are for a hectare of land and are given in Table 2. The growth rate of forage parameter (γ) used (0.1) is from Noy-Meir (1976) and represents a rangeland of high productivity. Most of Wyoming's rangelands would most likely not be classified as "highly productive" and we were unable to find any description of other potential parameters for this area, so the model also was solved for growth rates of 0.06 and 0.03 to determine how sensitive model results were to the productivity of forage. Following the work of Torell et al. (1991), weather is not explicitly stochastic, and the growth parameter is used to represent average productivity each year. This will allow a true steady state to emerge and initial sensitivity analyses to be performed. The parameter representing the maximum plant biomass for this area (V_m) of 350 kg on a dry matter basis is based on an estimate in Bastian et al. (2005) of 0.96 AUM $\cdot$ ha^{-1} productivity for Fremont County Wyoming (with an AUM representing 363 kg of grazeable forage). Huffaker and Wilen (1991) utilize daily animal consumption of 7.1 kg of dry matter per day over a grazing season taking an animal from 266 kg to 318 kg. Over a 120-d grazing season, this translated into 850 kg of dry forage consumption per animal. Huffaker and Wilen (1991), based on Noy-Meir (1976), also utilized an estimate of 20% of carrying capacity for the Michaelis Constant for consumption, translating here to 70.18. Without a better estimate for the Michaelis Constant, the model also was solved with values of 32.12 and 111.2 ($\approx$ 8.5% and 31.5% of carrying capacity) to evaluate how sensitive the outcomes are to this parameter.

Cattle prices (both initial and final) are based on the estimated price slide equation discussed previously. However, the model was also solved with cattle prices (both initial and final) increased, and likewise decreased, by 20% from mean

Table 2. Base scenario parameters used in the dynamic model.

Parameter	Value (alternative values used in sensitivity analysis)
γ (relative growth rate of forage)	0.1 · day^{-1} (0.06; 0.03)
V_m (maximum standing vegetation)	350 kg · ha^{-1}
C_m (maximum daily consumption)	7.1 kg · animal^{-1} · day^{-1}
V_r (mandatory forage residual)	0 kg · ha^{-1}
V_k (Michaelis Constant)	70.18 kg · ha^{-1} (32.12; 111.2)
W_i (initial weight)	250 kg
β (discount factor)	0.909091 (0.833333; 0.952381; 0.990099)
CC (carrying cost per animal)	$40.13 · animal^{-1}
Days on pasture	120
Death loss	2%

prices observed for comparison to the baseline outcome. The price of corn is based on mean values ($0.12 $\cdot$ kg^{-1}) from the LMIC data over the time period used in estimating the price function. The model also was solved for differing corn prices, specifically over the maximum ($0.166 $\cdot$ kg^{-1}) and minimum ($0.08 $\cdot$ kg^{-1}) prices observed in the LMIC data. Initial weight (250 kg) and days on pasture (120) are in line with a study done for the Wyoming Red Desert by Bastian et al. (1991) and results by Derner et al. (2008) of a long-term study in Wyoming. The grazing season is modeled from early June to early October. Van Tassell et al. (1997) calculated animal costs per AUM in a study including Wyoming. The sum of association fees, veterinary, moving, herding, miscellaneous labor and mileage, salt and feed, water, horse, and improvement maintenance costs from that study are $9.08 $\cdot$ AUM^{-1}. Inflating these animal costs to 2008 dollars results in animal costs of $14.33 $\cdot$ AUM^{-1}. This translates to animal carrying costs of $40.13 per head ($14.33 per AUM · 4 mo · 0.7 AUM per month) based on average animal weights over the season in this study. The discount rate used initially was 10%, but results were also generated for discount rates of 0.1%, 5%, and 20% for comparison. The model was solved using General Algebraic Modeling System (Brooke et al. 1998) to maximize the total of all discounted future returns to land (equation 6).

RESULTS

The model was initially solved using the baseline parameters (parameters not shown in parentheses in Table 2). For a hectare of land, given the initial parameters, the objective function converges to the long-run equilibrium of 195 kg of standing forage per hectare with an associated stocking rate of 1.66 head per hectare. The optimal solution converges to this amount of forage from starting points either above or below this amount. The results of our model are consistent with Noy-Meir's (1975) conclusions, which, ignoring economic consequences, state that the "safe carrying capacity" (p. 93) is defined as

$$H_S=\frac{\gamma V_K}{c_m} \qquad [8]$$

which is 0.988 head per hectare with the given base parameters.

Table 3. Sensitivity analyses and resulting steady state values across different parameters.

	Growth rate	Discount rate	Steady state values			
			State value[1] ($kg \cdot ha^{-1}$)	Stocking rate ($head \cdot ha^{-1}$)	End weight (kg)	Returns[2] ($\$ \cdot ha^{-1}$)
	0.1	0.01	196	1.6641	310	$112.25
		0.05	196	1.6661	310	$112.25
		0.1	196	1.6686	310	$112.25
		0.2	194	1.6735	310	$112.25
	0.03	0.1	193	0.5041	309	$33.66
	0.06		195	1.0035	310	$67.34
Corn price ($\$ \cdot kg^{-1}$)						
0.08	0.1	0.1	194	1.6740	310	$117.91
0.121			196	1.6686	310	$112.25
0.166			197	1.6620	310	$106.12
Michaelis Constant						
32.12	0.1	0.1	185	1.4569	319	$129.89
70.18			196	1.6686	310	$112.25
111.2			207	1.8404	303	$95.62
Output prices						
Decreased 20%	0.1	0.1	201	1.6417	310	$76.51
Base			196	1.6686	310	$112.25
Increased 20%			192	1.6821	309	$148.18

[1]Michaelis constant, 70.18; corn price, 0.121; growth rate, 0.1; discount rate, 0.1.
[2]2008 dollars.

Noy-Meir also shows that the maximum carrying capacity can be defined as

$$H_X = H_S + \frac{\gamma}{4c_m V_m}(V_m - V_K)^2 \quad [9]$$

which is 2.22 head per hectare given above base parameters. Given the above parameters, the maximum carrying capacity is approached but never realized, consistent with Noy-Meir's (1975) statement that a stocking rate just below this maximum capacity "may be a reasonable choice of 'normal' stocking in a commercial pasture" (p. 95). Moreover, these results are in line with other recommendations such as those found in Torell et al. (1989).

Table 3 compares outcomes across the differing parameter values utilized. Overall, it was economically optimal to leave over half of the standing vegetation. The parameters that had the greatest impact on ending standing forage were the Michaelis Constant and cattle prices. The variables with the greatest impact on stocking rate were the Michaelis Constant and the forage growth rate. The parameters with the greatest impact on financial returns were cattle prices and forage growth rate.

DISCUSSION

Optimal Steady State Values

Economically optimal ending forage values are around 196 kg of standing vegetation, except when the Michaelis Constant or cattle prices are varied, which caused optimal ending states to range from 185 kg to 207 kg and 192 kg to 201 kg, respectively. This implies that it is optimal for producers to leave just over half of the standing vegetation when considering future forage impacts due to current grazing where maximum standing vegetation is 350 kg. This would imply a lack of motivation for producers to overgraze rangelands, as long as previous assumptions about incentives to maximize land value hold. Even with a lower Michaelis Constant, it is not optimal to leave less than half of potential standing forage at season's end. Again, if the Michaelis Constant is interpreted as previously mentioned, a producer can improve returns to the land significantly through either carrying animals that are more efficient grazers (i.e., able to harvest more forage or gain more weight per unit of energy expended) or by improving the quality of the forage on the range, resulting in improved animal nutrition per unit of forage produced.

These results are in line with the traditional view of land managers to aim for 50% utilization of desired species as a general rule for range management for our study area (see for example Bastian et al. 1991). It should be noted, however, the "take half, leave half" rule of thumb has seen some criticism due to the fact that animals do not always graze on only the upper half of all plants (the criticism being not all parts of the plants are homogenous in terms of regrowth potential), as well as overlooking the possibility of vegetative changes that can result from these utilization guidelines (see for example Frost et al. 1994). Despite some limitations and criticisms, 50% utilization of key species has not been an uncommon goal of recommendations (although this recommendation can vary by both location and species). While obviously an oversimplification, the model assumes (see Noy-Meir 1975) the forage stand is composed of only one type of forage (the key species), with no difference in quality between plant parts. While Noy-Meir warns that results from his model may not be applicable to more complex systems, he does state that results would not be expected to be highly sensitive to deviations from his assumptions and should generally hold for situations that approximate those in his model.

When the forage growth parameter is 0.1, optimal long-run stocking rates are around 1.66 head per hectare. This is sensitive only to changes in the Michaelis Constant, varying between 1.45 and 1.84 head per hectare. This is due to the fact that per-animal consumption is determined by standing forage, but altering the Michaelis Constant ultimately alters the consumption per-animal at a given quantity of standing forage. Once consumption per animal is determined, the only way to remain at a steady state is to find the stocking rate that equates consumption to growth.

End weights for cattle were around 310 kg except where the Michaelis Constant was varied. With declining prices for higher sale weights, heavier animals did not optimize returns. As seen in Table 3, even with different Michaelis Constants, optimal sale weight does not exceed 320 kg.

If the forage growth parameter used is 0.06, the stocking rate drops to 1.0 head per hectare, and if the growth parameter is only 0.03, this falls to 0.5 head per hectare. Therefore, stocking rate depends on potential forage production, and optimal stocking rates should be aligned with this forage growth parameter.

Not surprisingly, returns per hectare are most responsive to changes in the forage growth parameter. Land with more forage production potential can carry more animals over the season to the same ending weights, resulting in much higher returns. The Michaelis Constant also has a large impact on return per hectare as well. Again, a producer with more efficient grazers or higher quality forage can produce more weight gain per hectare of land, resulting in higher returns to the land base. This is not only due to the ability to produce more gain per area of land, but the ability to do so with a lower stocking rate, resulting in lower variable costs per hectare.

Producers are also often aware of fluctuating prices. Indeed, according to this model, cattle prices have a large impact on returns per hectare. However, stocking decisions vary little across different cattle price levels, unlike the results of some previous modeling efforts (see for example Hart et al. 1988; Torell et al. 1989; Manley et al. 1997). Similarly, corn prices have an impact on financial returns per hectare (although less so than cattle prices), but optimal stocking decisions vary little. Unfortunately, this implies there is little a producer can do by means of grazing management to alleviate the impact of either low cattle prices or high corn prices.

Optimal Stocking Rate in Order to Achieve Steady State Values

A convenient outcome available with dynamic programming is the ability to determine the optimal response function, which prescribes stocking rate in this case. Given the desired steady state values described in Table 3, the optimal response function determines which stocking rate should be utilized in order to best reach the desired steady state (economically) for any given initial standing level of forage. In the beginning of the grazing season, a producer must make stocking decisions. Given that initial standing forage is observable, the approach utilized here allows a producer to make the stocking decision that will maximize total returns to land over the infinite horizon based solely on that standing forage level. Figure 1 shows what stocking rate should be set for various levels of initial standing forage across the different scenarios. In all cases, optimal stocking should be based on standing forage. When existing forage is not at the desired steady state, annual stocking should be lowered in order to reach the optimal ending forage levels. This implies that long-term productivity drives the stocking decisions more so than short-term profits.

Figure 1A illustrates that when utilizing an infinite planning horizon, the discount rate has little effect on optimal stocking rate for any given level of standing forage. As can be seen in Figures 1B and 1C, neither corn price levels nor steer price levels alter the optimal stocking rate for a given standing forage level. In fact, the reason Figures 1A–1C are shown in three dimensions is that in two dimensions, there is not any visible difference in optimal stocking rates for the alternative values of the parameters analyzed. However, Figures 1D and 1E show that growth rate, or plant productivity, and the Michaelis Constant alter optimal stocking patterns greatly for a given standing forage level. Given our model and its assumptions regarding maximization of the sum of discounted returns to the land, regardless of a producer's personal discount rate, the price level of corn, or the output price level, the optimal stocking rate is determined predominantly on standing forage for given biological response parameters. This is somewhat counter to previous studies such as Manley et al. (1997), which indicate high cattle prices can influence producers to temporarily increase stocking rates to potentially unstable levels.

As seen in Figure 1D, the difference in optimal stocking rate in terms of varying growth rate is greatest toward the center of the state space (i.e., the response surface across the state of the forage resource). Near an undisturbed range state (high amount of standing forage), optimal stocking is not as drastically different across forage productivity levels as producers' optimal response is to utilize the high amounts of standing forage. However, near states with lower standing forage, producers with less productive forage will need to stock at relatively lower rates in order to reach the desired ending forage stand. As the forage stand reaches the desired state, the difference in stocking is highest. Given the forage growth parameters used, this is where the forage is most productive.

As seen in Figure 1E, stocking rate differences are greatest toward a state with lower amounts of standing forage, given differing values for the Michaelis Constant. As the state of the forage is closer to maximum standing forage, the economically optimal stocking rates associated with the differing values of the Michaelis Constant converge. These results imply that a producer who is managing land that has low amounts of standing forage must stock at an even lower relative rate when faced with a system with a lower Michaelis Constant (whether that is due to either forage quality or the efficiency of grazing animals) in order to reach the desired steady state while maximizing long-term profits.

Obviously, a producer with more productive rangeland can set a higher stocking rate. It is interesting, however, that in a situation with a higher Michaelis Constant, whether through less productive grazers or lower quality forage, an economic optimum is achieved by stocking at a higher rate and ending

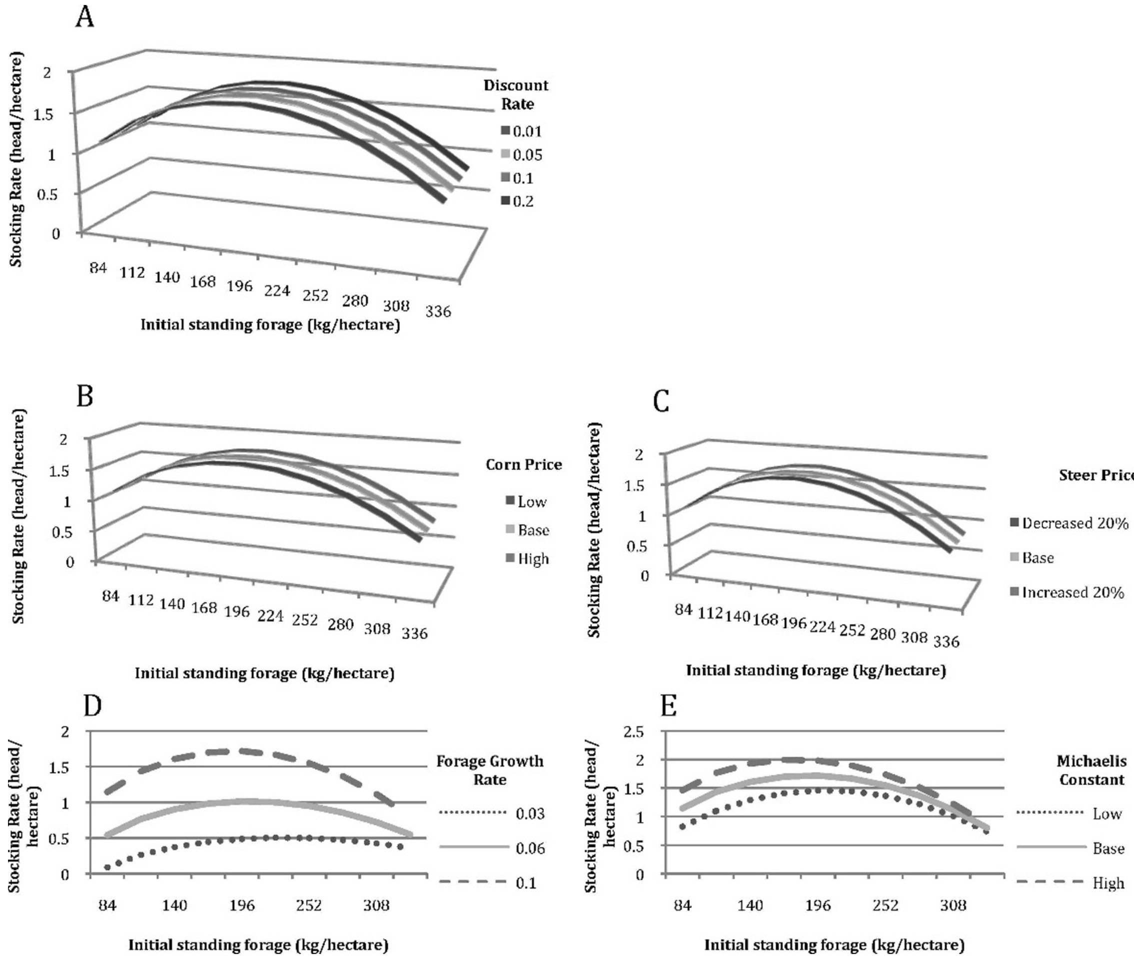

Figure 1. Economically optimal stocking rates given initial levels of standing forage across various values for the **A,** discount rate, **B,** corn price, **C,** steer price, **D,** forage growth rate, and **E,** Michaelis Constant.

with lower weight animals. Producers who are faced with a situation that relates to a lower Michaelis Constant, whether having more efficient grazers or higher quality forage, should stock at a lower rate and end up putting more weight on their animals. Again, this ultimately is due to the ability to allow more gain per animal while maintaining lower carrying costs associated with lower animal numbers.

IMPLICATIONS

Our model suggests that for stocker operations with the aim of maximizing the value of the land and with an infinite time horizon, it is optimal for producers to incur lower returns initially in order to improve rangeland health, as opposed to a producer interested in maximizing current year profits only. Although optimal levels of standing forage are reliant on growth rates of forage and consumption characteristics of animals, the idea of 50% utilization is fairly consistent with bioeconomically optimal stocking decisions from this model. In fact, in most cases, the economically optimal standing forage at season end is 55% of potential production. This result is consistent with previous studies investigating optimal stocking rates.

The results suggest that cattle and corn price levels can have a major impact on financial returns, but producers should not alter their stocking decisions based on variation in either of these price levels. Regardless of price levels, producers never had an incentive to overgraze the range in any of the scenarios evaluated for this model. Overall, the results indicate that a producer must be aware of current conditions of the range in order to make optimal decisions. Although selecting the proper stocking rate is vital to maintaining long-term range health, one of the largest impacts on financial returns for producers is to carry efficient grazers or have high quality forage, as evidenced by the sensitivity of the results to different values for the Michaelis Constant. This finding indicates that continued research by physical scientists related to animal performance and grazing efficiency as it relates to such variables as forage density and forage quality would improve knowledge about the Michaelis Constant and ultimately improve the ability of bioeconomic models, such as this, to prescribe optimal and sustainable grazing management strategies.

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