Prehistoric Patterns: A Mathematical and Metaphorical Investigation of Fossils

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While examining the delicate curves of a seashell or a gnarled oak branch, you may discover spirals, lines, or branching shapes—patterns of growth or simple structural observations. You may notice that the seashell looks like a snail shell, or that the oak branch resembles blood vessels, leading to connections you did not know existed. Fascinating, right?

Patterns and symmetry create a satisfying sense of order and unity, connecting us to the natural world. Drawing and observation help us explore this sense of order and our inherent attraction to pattern. We examine an object's properties and recreate them on paper to learn more about the object and the world around us. For example, Leonardo DaVinci's drawings of plants that investigated spiral growth aimed to "understand the forces and processes underlying their forms" to connect morphology and physiology.[1] Drawing is a way of seeing the world and learning from it, applying what we see to our own lives.

Leonardo DaVinci's observational drawings, which investigated the connections between nature and mathematics informed my investigation of the hidden mathematical patterns in fossils. By drawing fossils and analyzing their properties, I found that they are mathematically significant and combine the biological world with the mathematical one. Through drawing, I discovered that ammonites imitate the famed Fibonacci sequence and connect to modern landscape mapping. I also discovered that fish fossils contain symmetry and fractals in their fins that relate to modern fields of Physics and Mathematics such as Chaos and Group theory. Finding the mathematical significance of fossils through drawing integrates patterns, symmetry and beauty with a new understanding of an object to connect personal perceptions with the natural world.

Introduction: The Beauty of Symmetry

Symmetry is a form of beauty. Beauty, as defined by Plato, "is always an aspect

1 Capra, Fritjof, "Learning from Leonardo" (San Francisco: Berrett-Koehler Publishers, 2014) 3.

of the good," and "in the beautiful and good, proportion is involved."[2] What Plato was referring to in "proportion" was symmetry; it implies good proportions and therefore is an aspect of beauty. Symmetry creates "themes of proportion…used by Greek and Gothic architects"[3] to amaze and inspire those who see the finished product and admire its beauty. Additionally, the mental aspect of the appreciation of symmetry or pattern is "that of perceiving relationships."[4] We, the viewers, look at a fossil of a fish, and see symmetry in its backbone. By closer inspection, we also see through this symmetry the relationship between fin and torso, bone and branch, and form and function. The aesthetic pleasure comes from realizing the "interconnecting paths"[5] of symmetries, patterns, and mathematical theorems. For example, ammonites, ancient nautilus-like creatures that swam in the oceans near Madagascar, have similar shell patterns to modern snails, and both patterns follow the Fibonacci sequence. Fish fossils display fractal patterns that resemble tree branches and hyperbolic graphs. Overall, these interconnecting paths show the relationships between mathematics and nature that have been preserved for millions of years. They show beauty in time, beauty in creation, and beauty in calculation and investigation.

Fossils and Fibonacci: Mathematical Patterns in Ammonites

Ammonites have mathematical proportions and patterns within their shells. Through drawing, I found that the most obvious mathematical pattern in an ammonite shell is the Fibonacci sequence, a series of numbers developed by the mathematician Leonardo Bonacci in the 13th century. The Fibonacci sequence is a repeating pattern of numbers, represented by the equation $x_{n+1} = x_n + x_{n-1}$ for $n > 1$. The base for this recursive definition is $x_0 = 0$ and $x_1 = 1$, and so, the following numbers in the recursive pattern will be $x_2 = 1$, $x_3 = 2$, $x_4 = 3$, $x_4 = 5$, $x_5 = 13$, and so on. Each number in the sequence is the sum of the two numbers before it. In Figure 1, this pattern

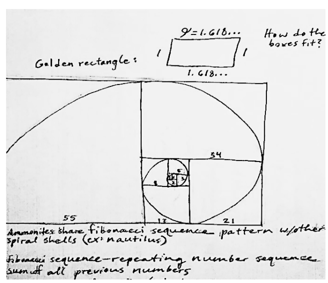

Figure 1: Ammonites share the Fibonacci sequence (a number sequence involving the sums of previous numbers) with other spiral shells. This physical and numerical pattern has appeared continuously in nature throughout time.

2 Hon, Gloria, "From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept" (Pittsburgh: Springer, 2008) 94.

3 Ghyka, Matilda, "The Geometry of Art and Life" (New York: Sheed and Ward, 1946) xi.

4 Huntley, H.E., "The Divine Proportion; A study in Mathematical Beauty" (New York: Dover Publications, 1970) 118,143.

5 Ibid.

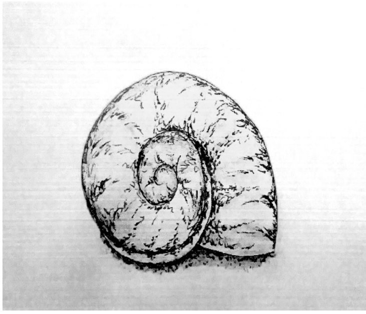

Figure 2: Outward spiraling of ammonite shell

of numbers appears in the ammonite shell as it spirals out. Rendering boxes that follow the sequence creates a logarithmic spiral. The shell spirals out continuously, so there is a continuous addition of width (Figure 2).

Investigating ammonite fossils by drawing helped me find the Fibonacci sequence in not only ancient organisms but also modern mapping. The Moiré pattern, which is used to map landscapes, derives from the Fibonacci sequence. The ammonite imitates the spiral of the Moiré pattern, in which two textures with black and white components are placed on top of each other by "forming the union or intersection of the black pointsets."[6] The Moiré pattern creates a new perspective on contour maps and is a visual representation of applied mathematics (Figure 3). Therefore, drawing reveals a relationship between the Fibonacci spiral of an ammonite and mathematical landscape mapping.

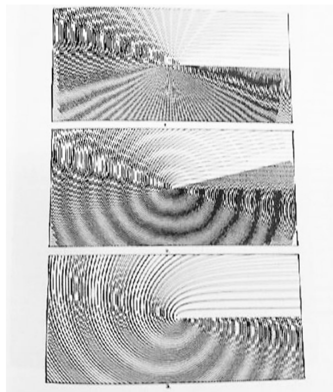

Figure 3: Fibonacci's spiral in the Moiré pattern

The logarithmic spiral of the Fibonacci sequence creates an "immediate appeal to the eye,"[7] and also has mathematical appeal. The sum of the series of segments within the Fibonacci spiral adds up to "precisely Phi times OA,"[8] where Phi is given as $\varphi = 1.6180...$, and OA is $\varphi^0 = 1$, as seen in Figure 4.

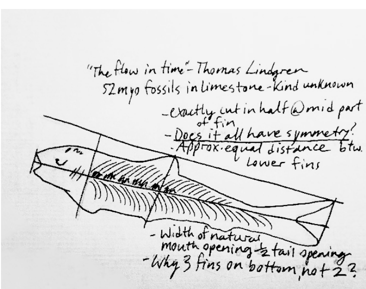

Figure 4: Phi in the ammonite shell (Huntley)

Phi represents the "Golden Ratio," a number that appears in many natural objects, such as pinecones and broccoli. The Golden Ratio and logarithmic spiral represent the growth of organic life as "gravitation is held to prevail in the physical world."[9] If the Golden Ratio or a logarithmic spiral represent "ideal growth," they

6 Pickover, Clifford "The Pattern Book: Fractals, Art and Nature" (Singapore: World ScientificPublishing, 1995) vii-46.

7 Huntley, H.E., "The Divine Proportion; A study in Mathematical Beauty" (New York: Dover Publications, 1970) 67,68.

8 Ibid.

9 Cook, Sir Theodore Andrea "The Curves of Life" (New York: Dover Publications, 1979) 6.

could provide insight into the commercial production of crops, the innerworkings of plants, and the growth of organisms that died out millions of years ago.

Fractals in Fish: Connections to Mathematics and Physics

Fish fossils provide fascinating mathematical insights into fractals, which are intertwined with Chaos theory and Group theory. While sketching the fish, I noticed that each fin branched out continuously. Each bone of the fossilized fish's fin divides once, then twice, then thrice from its origin, to form a pattern seen in tree branches and blood vessels. This branching pattern resembles a fractal, a mathematical and visual pattern that infinitely repeats itself. Fish fossils contain naturally-occurring fractals, but fractals also exist in modern computer design and art. The drawings shown in Figures 5 and 6 render tree-like patterns that imitate computer-generated fractals (Figure 7).

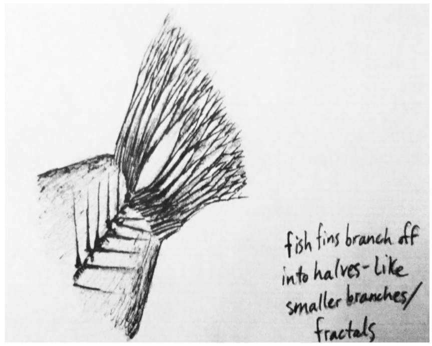

Figure 5: Bones in fossilized fish fins divide into halves, then smaller branches.

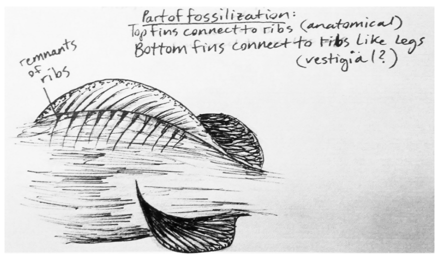

Figure 6: Fins branch like trees into fractal patterns

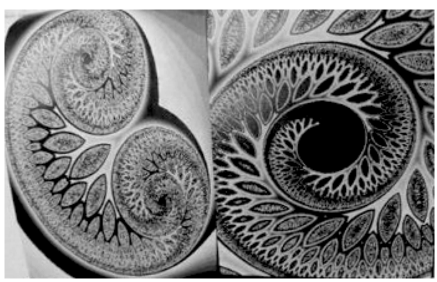

Figure 7: Computer-generated fractals

Fractals, while not only describing similarities in shape between different organisms, such as the fins of fish and tree branches, also apply to modern physics and mathematics. Chaos theory, the study of how small changes in the universe have large impacts, and fractal geometry, "go hand in hand." Fractals and Chaos Theory incorporate "intricately-shaped objects,"[10] and the fact that "chaotic processes often produce fractal patterns." Investigation of fish fins through drawing leads to new insights into patterns within them and their relationship to mathematics.

The patterns and symmetry found in fish fossils also connect to the

10 Pickover, Clifford "The Pattern Book: Fractals, Art and Nature" (Singapore: World Scientific Publishing, 1995) viii.

mathematical field of Group theory, which investigates visual symmetry in relation to groups. A group consists of elements that have algebraic operations applied to them. A group $(G,*)$ consists of a set G and a binary operator $*$ defined on G (a calculation that involves an order pair from $G \times G$, such as multiplication or division of two numbers from a number system) defined so that the following four rules are true:[11]

1. If a and b are elements of the set G (i.e. $a, b \in G$), then $a * b$ is also an element of G ($a, b \in G$).
2. Using the associative property, if a, b, and c are elements of G ($a, b, c \in G$), then $(a * b) * c = a * (b * c)$.
3. G contains an identity e with respect to the operation $*$, so that for every $x \in G$ (x as an element of G), we have $e * x = x * e = x$. For example, 0 is an identity, so adding 4 and 0 means 4+0 = 0+4 = 4.
4. Inverses exist for values within G.

In group theory, mapping groups and their elements creates patterns. For example, a hyperbolic group generates images akin to fractals when mapped on a vector space. The fractal-like nature of hyperbolic groups mimic the symmetries found in the fish fossils (Figure 8), providing insight into group theory and the mathematical definitions of symmetry.

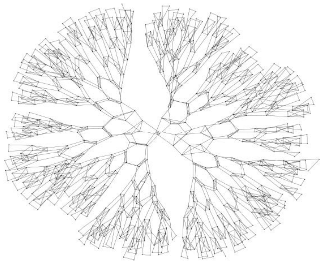

Figure 8*: Hyperbolic set symmetry (Feydy-ENS Canche)*

Through the Eyes: The Appeal and Aesthetics of Patterns and Symmetry

Investigation of Fibonacci sequences in ammonites and fractals in fish fins through drawing explores the inherent appeal of patterns and symmetry. Patterns and symmetry appear in art, architecture, and nature. Patterns in nature are especially important in art, as modern artists have a "growing fascination with symmetry and repetition in design."[12] Therefore, the Fibonacci spiral and fractals seen in fossils appeal to a wide audience.

Additionally, drawing natural patterns and symmetries is aesthetically pleasing to the artist. It is satisfying for the eye to render an object with a perfect balance of shape and geometry that can fit inside familiar figures (Figures 9 and 10). An ammonite's shell splits nearly equally into quadrants, and each chamber of the shell radiates to the outer edge, creating a perfect circle in the center.

11 Kennerly, Sam "A crash course in Group Theory" (Mathematic Journals, 2010) 3.

12 Pickover, Clifford "The Pattern Book: Fractals, Art and Nature" (Singapore: World Scientific Publishing, 1995) vii.

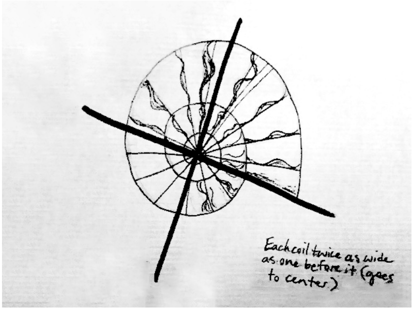

Figure 9: The internal chambers of an ammonite shell appear to start from a single origin point and radiate outward with nearly even width.

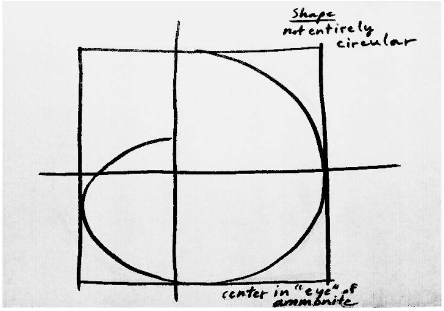

Figure 10: Curvature and symmetry: The ammonite's overall shape is not entirely circular, but it can fit inside symmetrical geometric shapes such as rectangles.

Symmetry and pattern have a special place in aesthetic pleasure. Symmetry, defined as the "exact correspondence of a form,"[13] appeals to the creator in drawing, and to the viewer in viewing.

As the creator of a drawing, symmetrical shapes entice me, because they create a feeling of perfection and unity. While rendering the form of a fish fossil (Figure 11), the symmetry present in the vertebrae and the curves of the body fascinates me. The spinal cord in this particular fossil cut the distance from the top of the dorsal fin to the belly precisely in half. The apex of the dorsal fin perfectly divides the fish vertically. Looking at the proportions and symmetries of the fish while drawing it helped me gain a holistic perspective of its form through drawing.

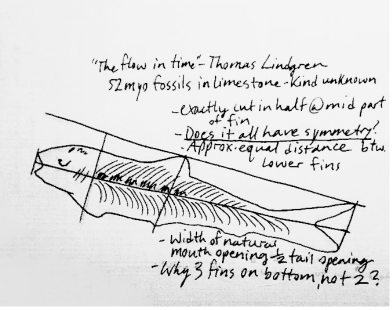

Figure 11: Symmetry in fish fossils- width of mouth equals ½ width of tail aperture, some fossils can be divided in half precisely from tip of dorsal fin.

Through the practice of artists such as Leonardo DaVinci, observational drawing leads to understanding. The physical acts of drawing and careful observation allow both the artist and viewer to find the Fibonacci sequence in ammonites and fractals in fish fossils and relate them to modern mathematics. Connecting two seemingly unrelated things through drawing, mathematics and fossils, creates a new form of understanding and a new form of unity. When combined with the philosophy of beauty and symmetry's role in aesthetics, both artist and viewer gain insight into different fields of study and the natural world itself.

13 Field, Michael "Symmetry in Chaos." (Philadelphia, Society for Industrial and Applied Mathematics:2009) 3.